Fitting Bayesian time series models

FISH 507 – Applied Time Series Analysis

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Overview of today's material

- Quick overview of Stan
- Manipulating and plotting Stan output
- Examples of time series models

Review of models we've used so far

Models

- Regression
- ARMA models
- State Space Models
- Dynamic Factor Analysis
- Dynamic Linear Models
- MARSS models (multivariate time series models)

Why Bayesian?

- Complex hierarchical models
  - Non-linear models
  - Hierarchical or shared parameters
  - Non-normal data
- Inference: what's the probability that the data are less than some threshold?
- No bootstrapping!
  - We get credible intervals for parameters and states simultaneously

Using STAN

What is Stan?

- Powerful, cross-platform and cross-language (R, Julia, Matlab, etc) that allows users to write custom code that can be called directly from R
- Estimation can be fully or approximate Bayesian inference, or maximum likelihood optimization (BFGS)
- Useful links:
  - Stan homepage
  - Stan manual
  - rstan

Options for using Stan in this class

- Write your own code (based on examples in the manual, etc)
- Use an existing package
- Use our bundled code to get started with simple models (we'll start here)

Existing packages: rstanarm and bmrs

- Both packages very flexible, and allow same syntax as basic lm/glm or lmer models, e.g.

```
rstan::stan_lm
rstan::stan_glm
rstan::stan_glmer
```

- Vignettes brms rstanarm

Potential limitations

`brms` includes autocorrelated errors, non-normal data, non-linear smooths, etc.

- But doesn't allow for AR processes on parameters / latent variables
- Also doesn't include multivariate data

Advantages

brms offers notation that should be very familiar to run many classes of models,

```
brms::brm(y ~ x * z + (1|group), data=d)
brms::brm(y01 ~ x * z + (1|group), data=d, family = binomia
brms::brm(bf(y ~ s(x)), data=d)
```

To install code for this class

- We'll need to install these packages to run Stan,

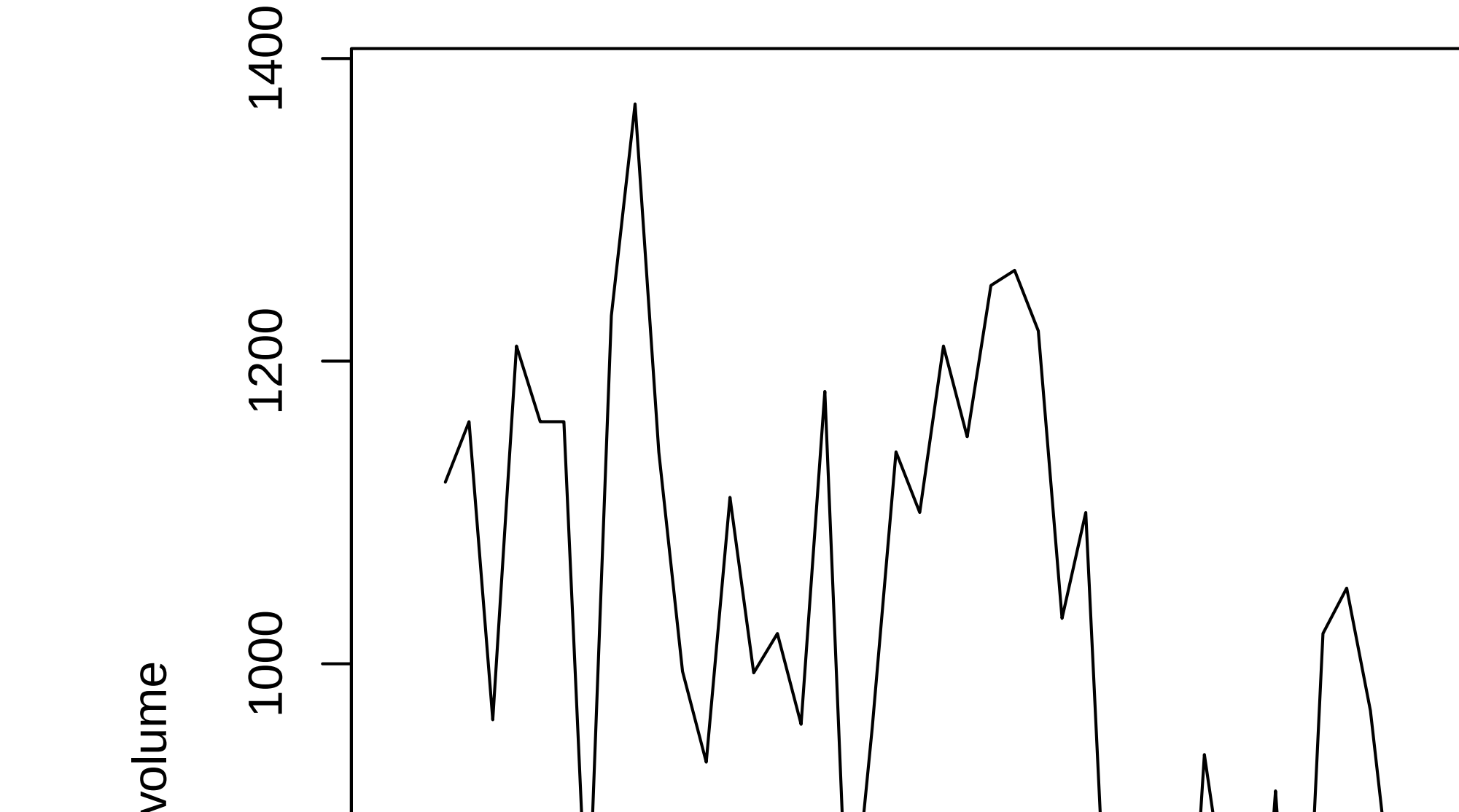
```
install.packages("rstan", repos = "https://cloud.r-project
install.packages("devtools", repos = "https://cloud.r-proje
```

- And then we can install our custom package for the class with bundled Stan time series models

```
devtools::install_github(repo="nwfsc-timeseries/atsar")
install.packages("atsar")
```

Working with Stan output

- We'll start with a simple example, using a classic dataset of flow on the Nile River



Working with Stan output

Fitting a regression model to this data doesn't make a lot of sense, but it will introduce us to the basic functionality of our wrapper functions. We'll start with a function named `fit_stan`

```
library(atsar)
lm_intercept = fit_stan(y = as.numeric(Nile),
                        x = rep(1, length(Nile)),
                        model_name = "regression")
```

Working with Stan output

The output of the fitted model can be examined a number of ways. Starting with the simple summaries,

```
lm_intercept
```

This model ran 3 MCMC chains (the default) with warmups of 500, followed by 500 more iterations. These latter were stored (so 1500 parameter samples total)

Working with Stan output

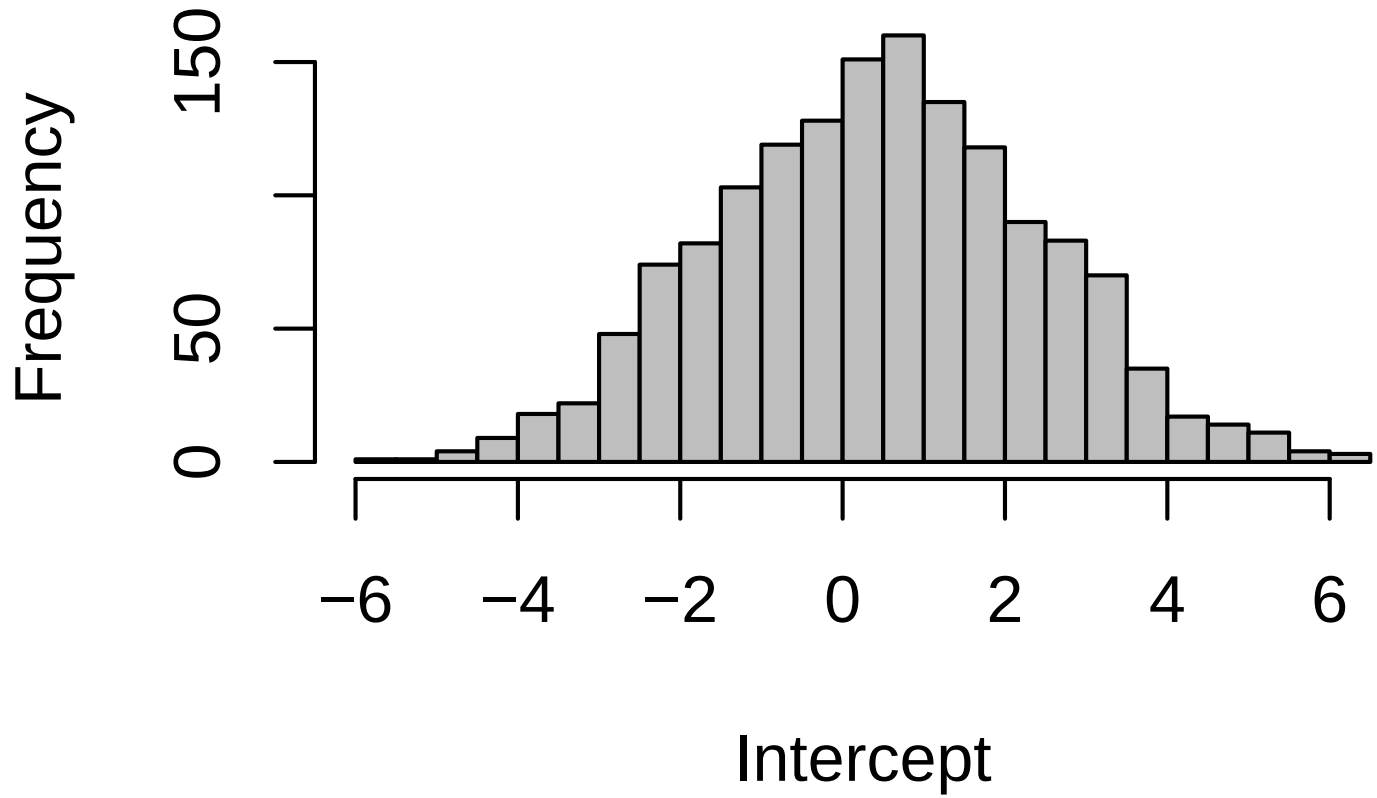
But we're probably more interested in the values for individual parameters. We can pull these out in a few different ways, and plot them with either base graphics or ggplot. As a first option for getting values we can manipulate, we can use the `extract` function,

```
pars = rstan::extract(lm_intercept)
```

Plotting with Stan output

Then we can do all kinds of things with this output, like making a histogram

```
hist(pars$beta, 40, col="grey", xlab="Intercept", main="")
```



or calculate summary statistics

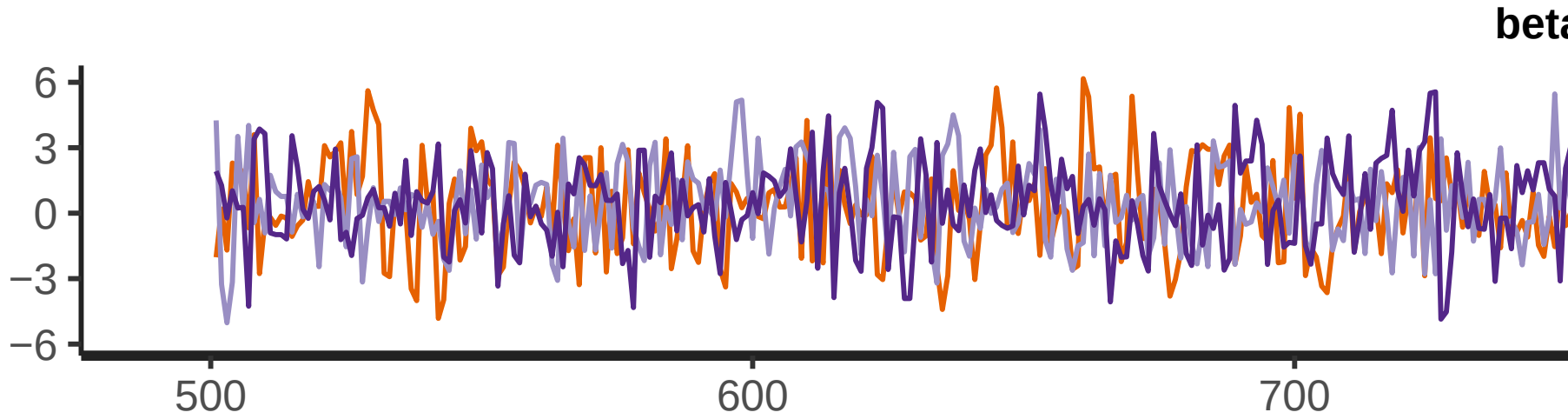
```
quantile(pars$beta, c(0.025,0.5,0.975))
```

Plotting with Stan output

The object `lm_intercept` is a `stanfit` object, which means there's a lot of other plotting functionality from the rstan package we can use

- First, let's make some traceplots

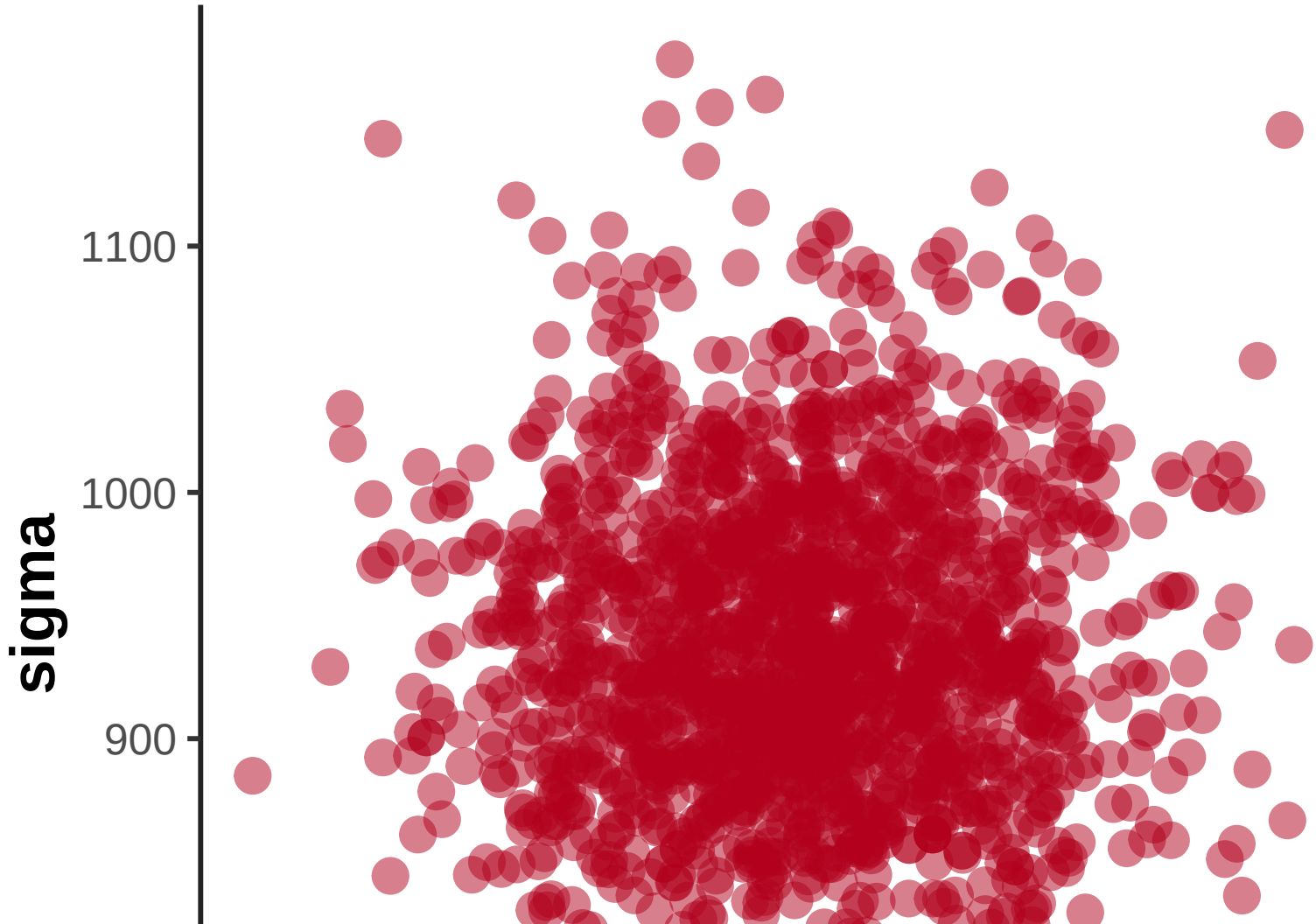
```
rstan::traceplot(lm_intercept,
                 pars = c("beta[1]","sigma"), nrow=2, ncol=
```



Plotting with Stan output

- Second, we can examine the correlation between parameters with a pairs plot,

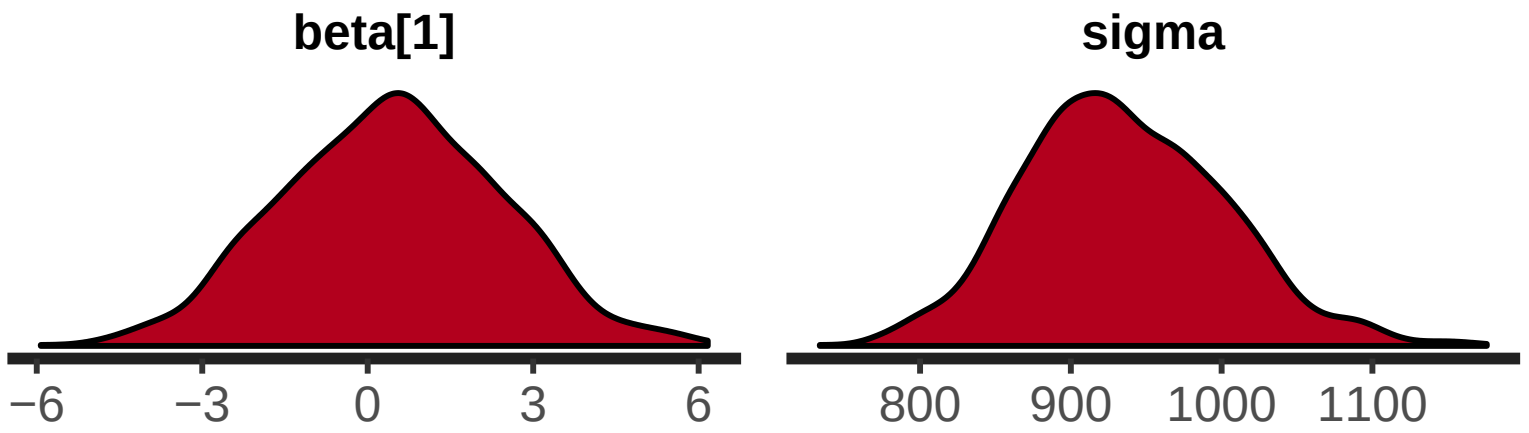
```
rstan::stan_scat(lm_intercept, pars = c("beta[1]","sigma"))
```



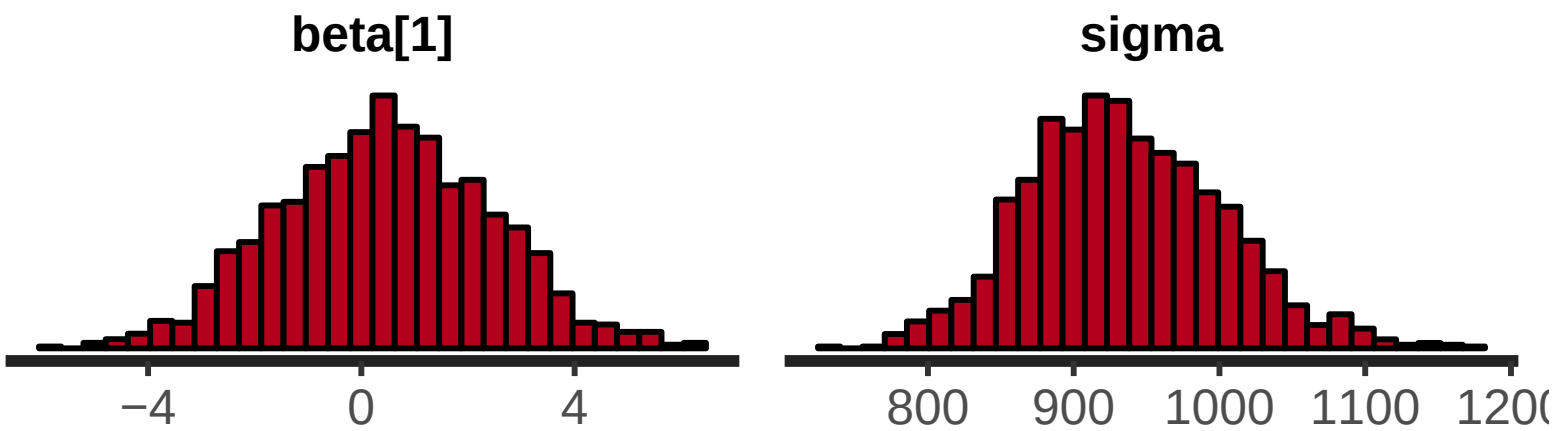
Plotting with Stan output

- Third, maybe we want to make some density plots or histograms

```
rstan::stan_dens(lm_intercept, pars = c("beta[1]","sigma"))
```



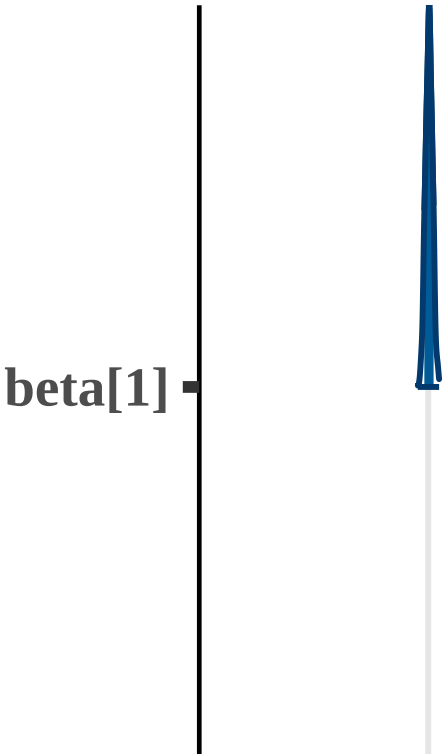

```
rstan::stan_hist(lm_intercept, pars = c("beta[1]","sigma"))
```



Plotting with Stan output

- Fourth, we can look at implementing some of the plots included in bayesplot

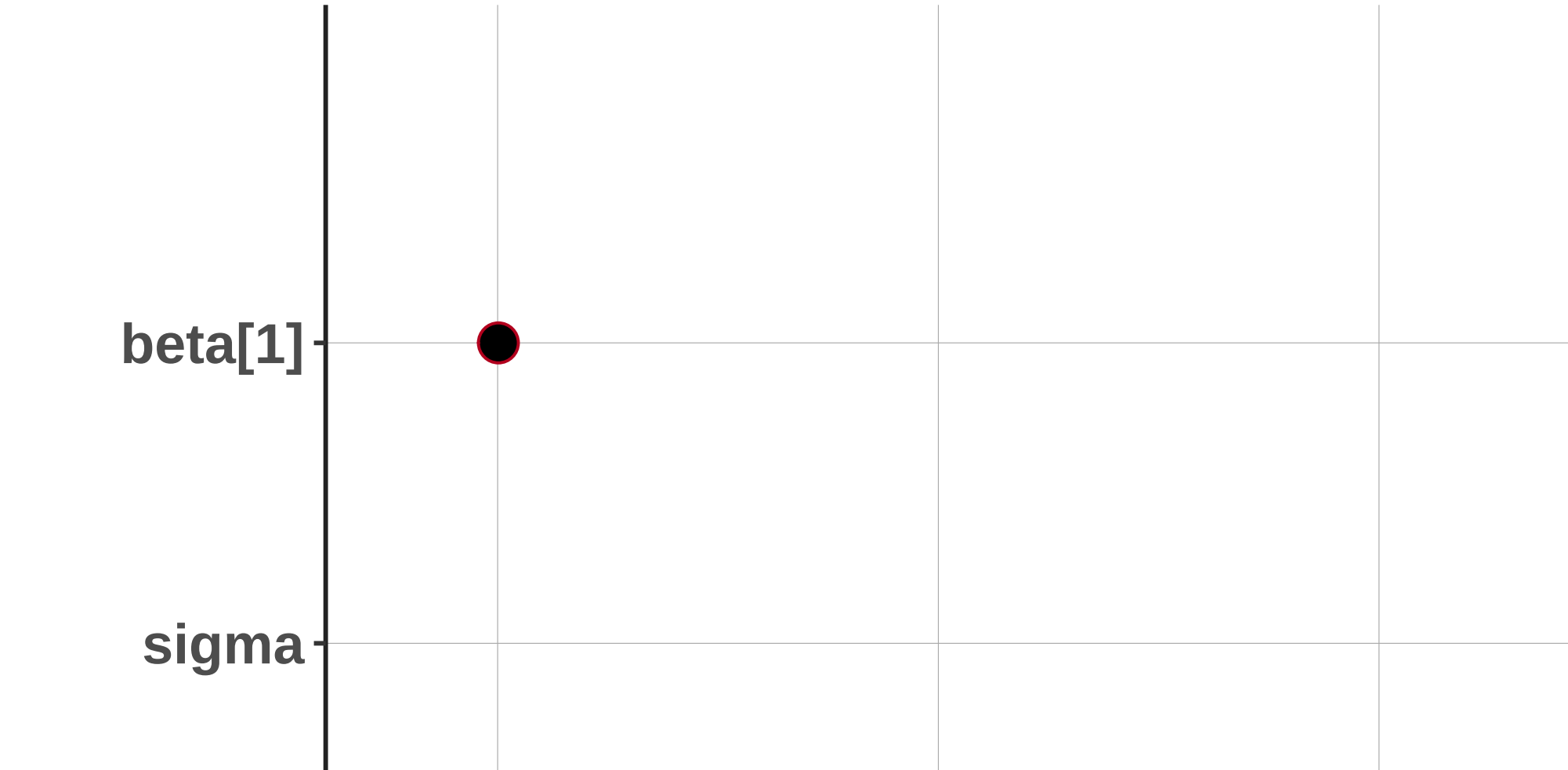
```
library(bayesplot)
mcmc_areas(as.matrix(lm_intercept),
           pars = c("beta[1]","sigma"),
           prob = 0.8)
```



Plotting with Stan output

- Another way to show the same uncertainties is with dotplots and credible intervals,

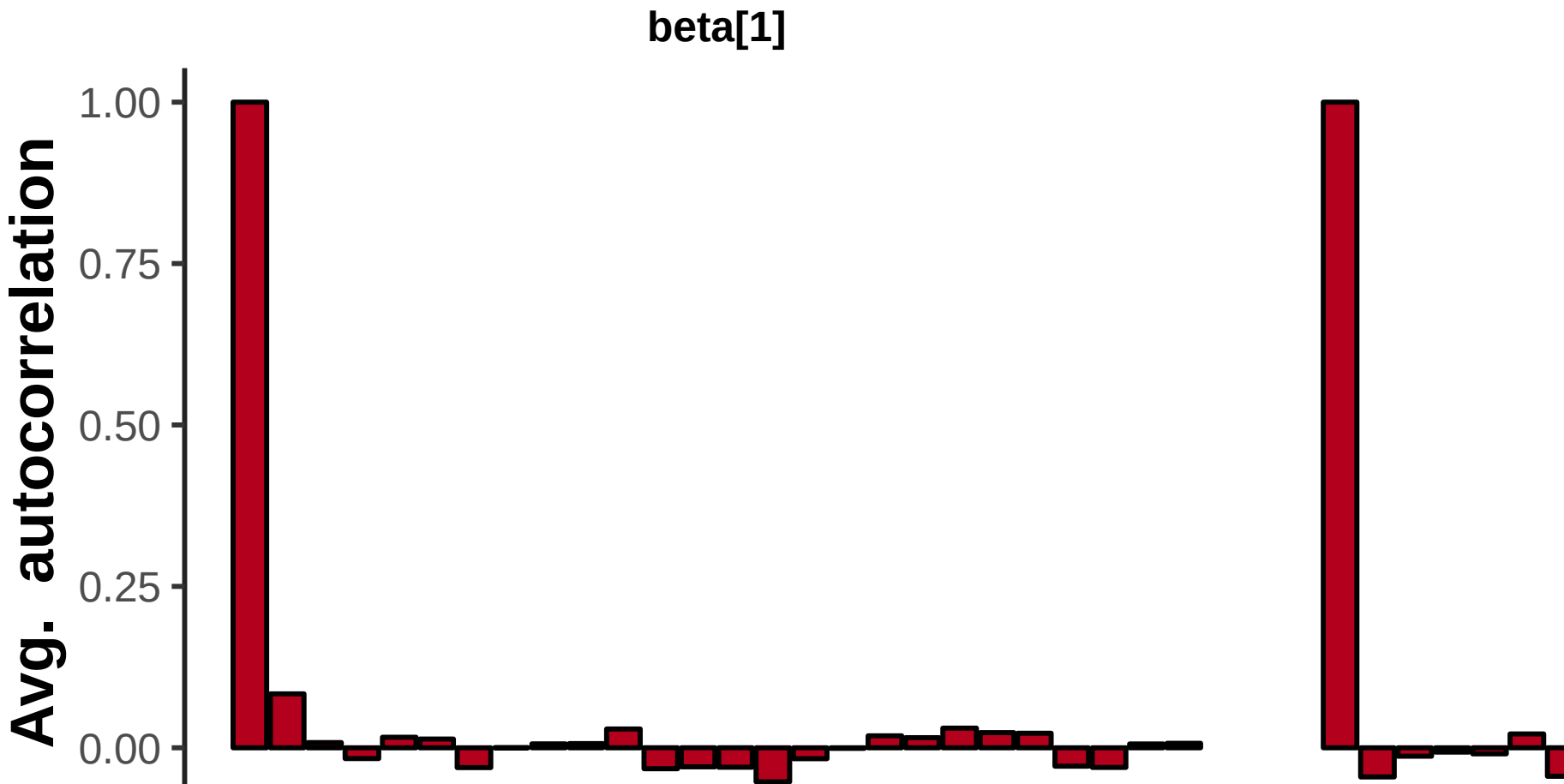
```
rstan::stan_plot(lm_intercept, pars=c("beta[1]","sigma"))
```



Plotting with Stan output

- We can also look at the autocorrelation of each parameter, which is another useful diagnostic for convergence (we want very low levels of autocorrelation)

```
rstan::stan_ac(lm_intercept, pars=c("beta[1]","sigma"))
```



Plotting with Stan output

These plots only the tip of the iceberg for plotting. For more great examples of the kinds of plots avaialable, see these vignettes:

- Examples on Stan
- Jonah Gabry's introduction to bayesplot
- Matthew Kay's introduction to bayesplot and tidybayes

Tidy summaries from Stan output

Using the `broom` package, we can also extract some tidy summaries of the output

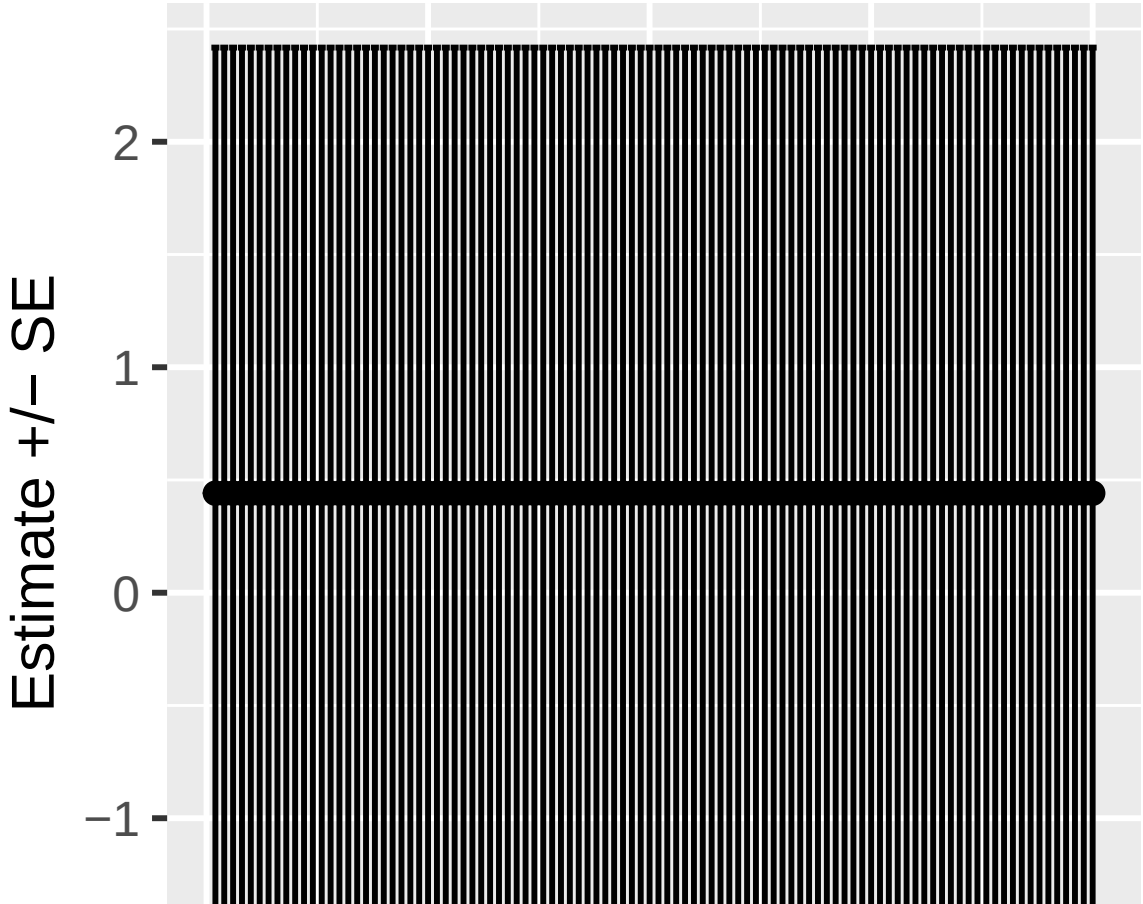
```
coef = broom::tidy(lm_intercept)
head(coef)
```

```
## # A tibble: 6 x 3
##   term    estimate std.error
##   <chr>      <dbl>     <dbl>
## 1 beta[1]    0.441      1.98
## 2 sigma    935.        66.9
## 3 pred[1]    0.441      1.98
## 4 pred[2]    0.441      1.98
## 5 pred[3]    0.441      1.98
## 6 pred[4]    0.441      1.98
```

Tidy summaries from Stan output

These tidy summaries can then be fed into ggplot for example

```
coef = broom::tidy(lm_intercept)
ggplot(coef[grep("pred",coef$term),], aes(x = 1:100,y=estim
  geom_point() + ylab("Estimate +/- SE")+ xlab("")+
  geom_errorbar(aes(ymin=estimate-std.error, ymax=estimate-
```



Preserving chain order

For models with multiple chains, we might want to preserve the chain ID to look at individual chain diagnostics. Remember that

- Each chain is independent
- `extract` defaults to merging samples from all chains together, e.g.

```
extract(object, pars, permuted = TRUE)
```

- But summaries can be generated for each combination of parameters-chains by setting

```
extract(object, pars, permuted = FALSE)
```

More time series models: random walk

This model should be familiar,

$$E[Y_t] = E[Y_{t-1}] + e_{t-1}$$

- We'll fit model to temperature data
- Note that the use of the argument `model_name` and `est_drift`. By not estimating drift, we asssume the process is stationary with respect to the mean

```
data(airquality)
Temp = airquality$Temp # air temperature
rw = fit_stan(y = Temp, est_drift = FALSE, model_name = "rw
```

More time series models: random walk

Did the model converge?

- One quick check is to look at the value of R-hat for each parameter (generally should be small, < 1.05 or smaller)

```
rw_summary <- summary(rw, pars = c("sigma"),
                      probs = c(0.1, 0.9))$summary
print(rw_summary)
```

```
##              mean     se_mean        sd       10%       90%
## sigma 5.764972 0.01281503 0.328861 5.358552 6.205832 658
```

More time series models: univariate state space models

State equation:

$$x_t = \phi x_{t-1} + \varepsilon_{t-1}$$

where $\varepsilon_{t-1} \sim Normal(0, q)$

Observation equation:

$$Y_t \sim Normal(x_t, r)$$

- Let's compare models with and without the AR parameter ϕ in the process model

More time series models: univariate state space models

We can first run the model with ϕ,

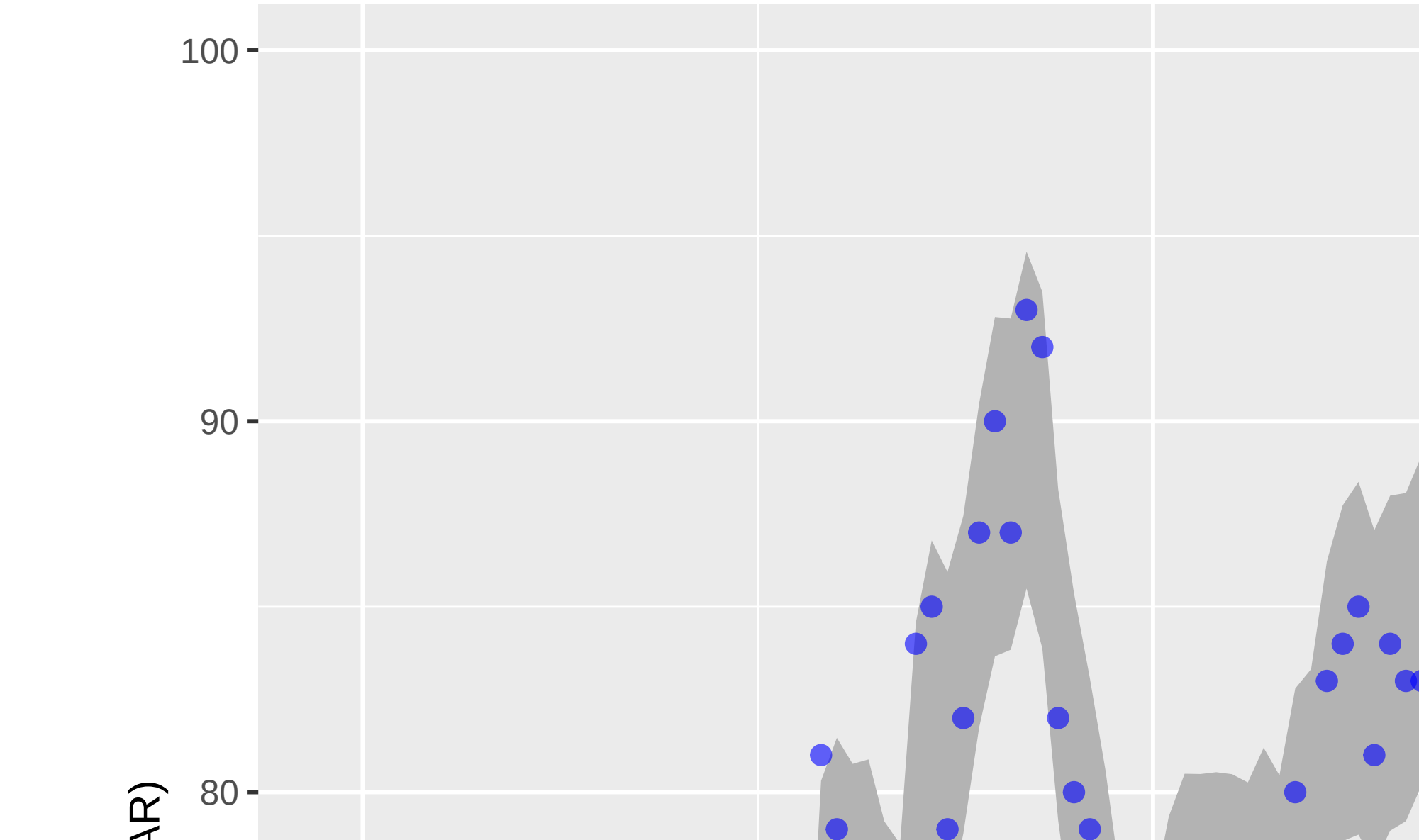
```
ss_ar = fit_stan(y = Temp, est_drift=FALSE, model_name = "s
```

then without,

```
ss_rw = fit_stan(y = Temp, est_drift=FALSE, model_name = "s
```

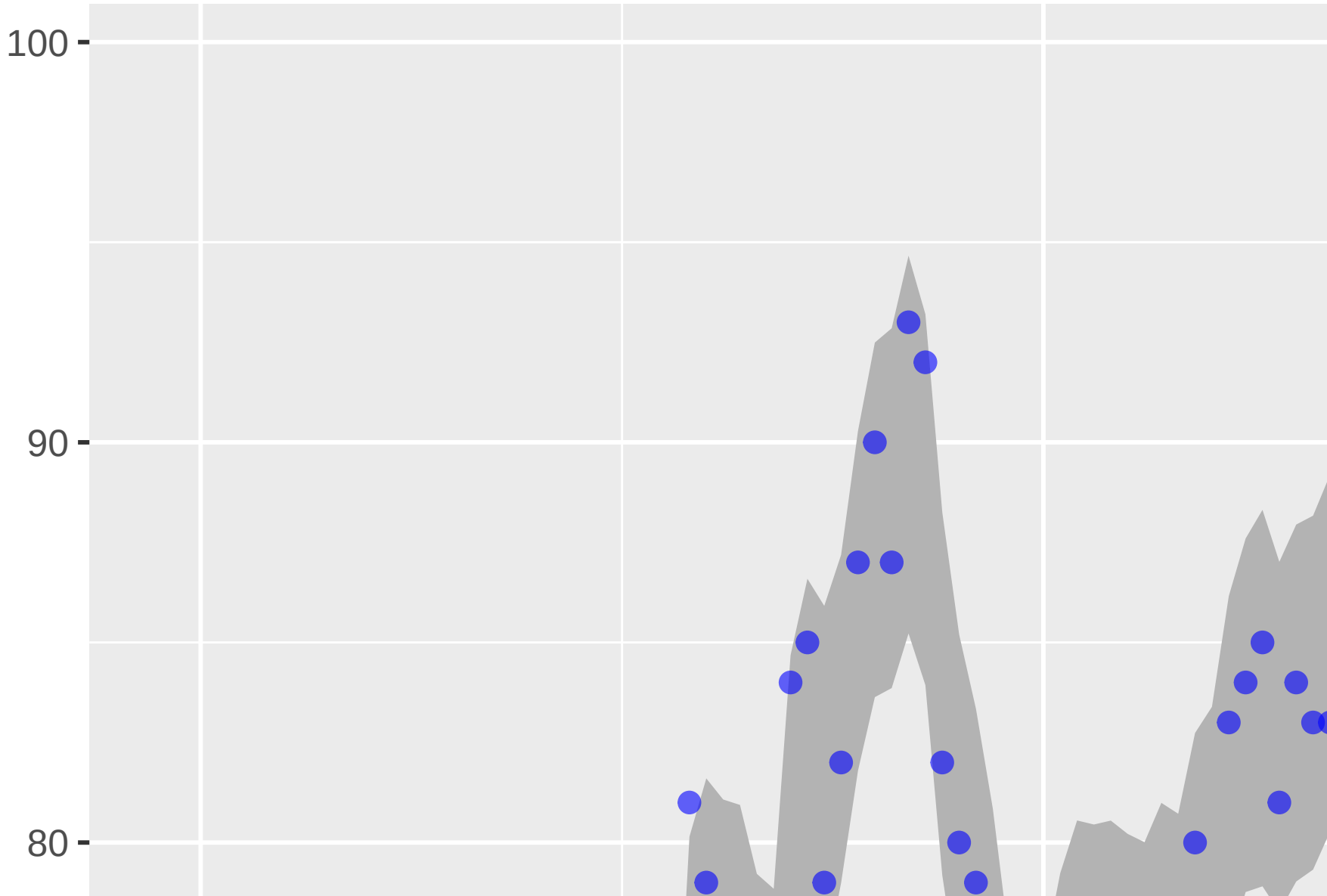
More time series models: univariate state space models

Estimates from the AR(1) state space model



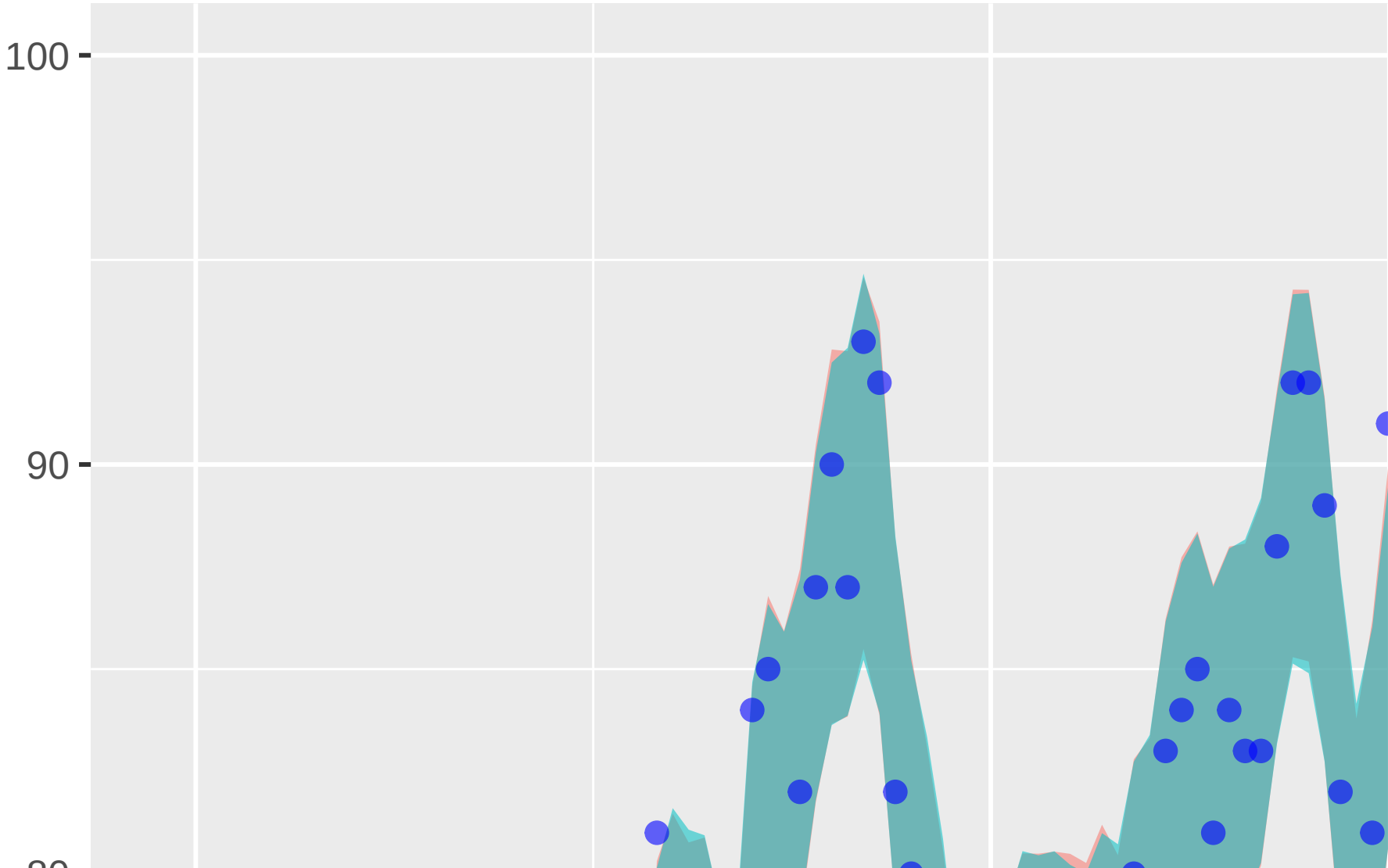
More time series models: univariate state space models

Estimates from the RW state space model



More time series models: univariate state space models

Estimates from both models (note the difference in credible interval widths)

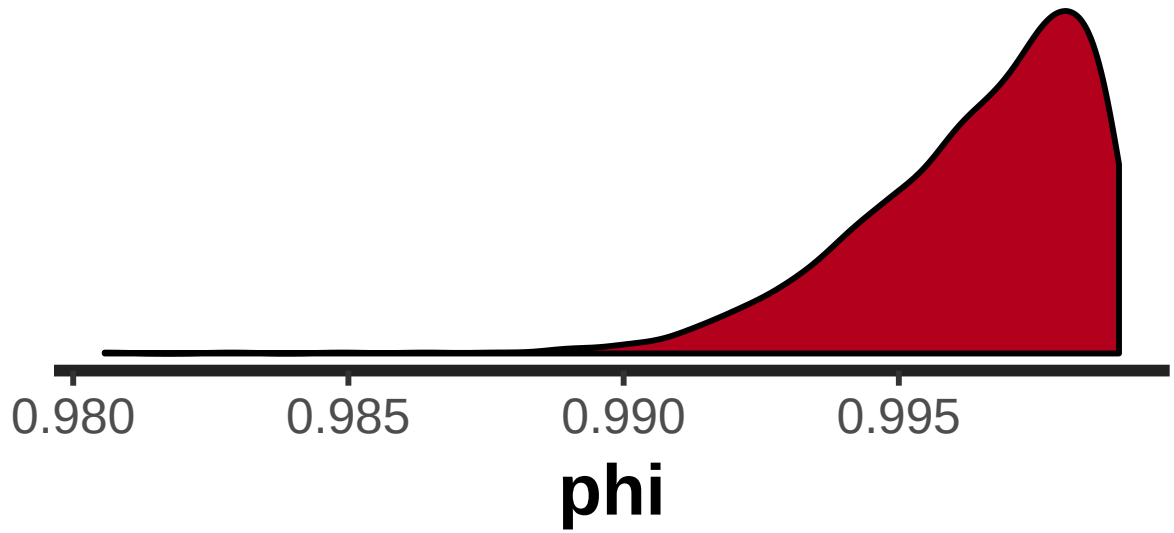


More time series models: univariate state space models

We might be also interested in looking at posterior estimates for these models.

- What is the posterior distribution of ϕ?

```
rstan::stan_dens(ss_ar, c("phi"))
```



- This shows with a value near 1, the behavior of the model should be very similar to the random walk (from the predictions, it is!). The multi-modal distribution probably is an indicator of convergence issues

More time series models: univariate state space models

We might also be interested in derived quantities.

- For example, what's the probability of temperature exceeding 90 degrees?

```
pars = extract(ss_ar)
p = length(which(pars$pred > 90)) / length(pars$pred)
print(round(p,3))
```

```
## [1] 0.077
```

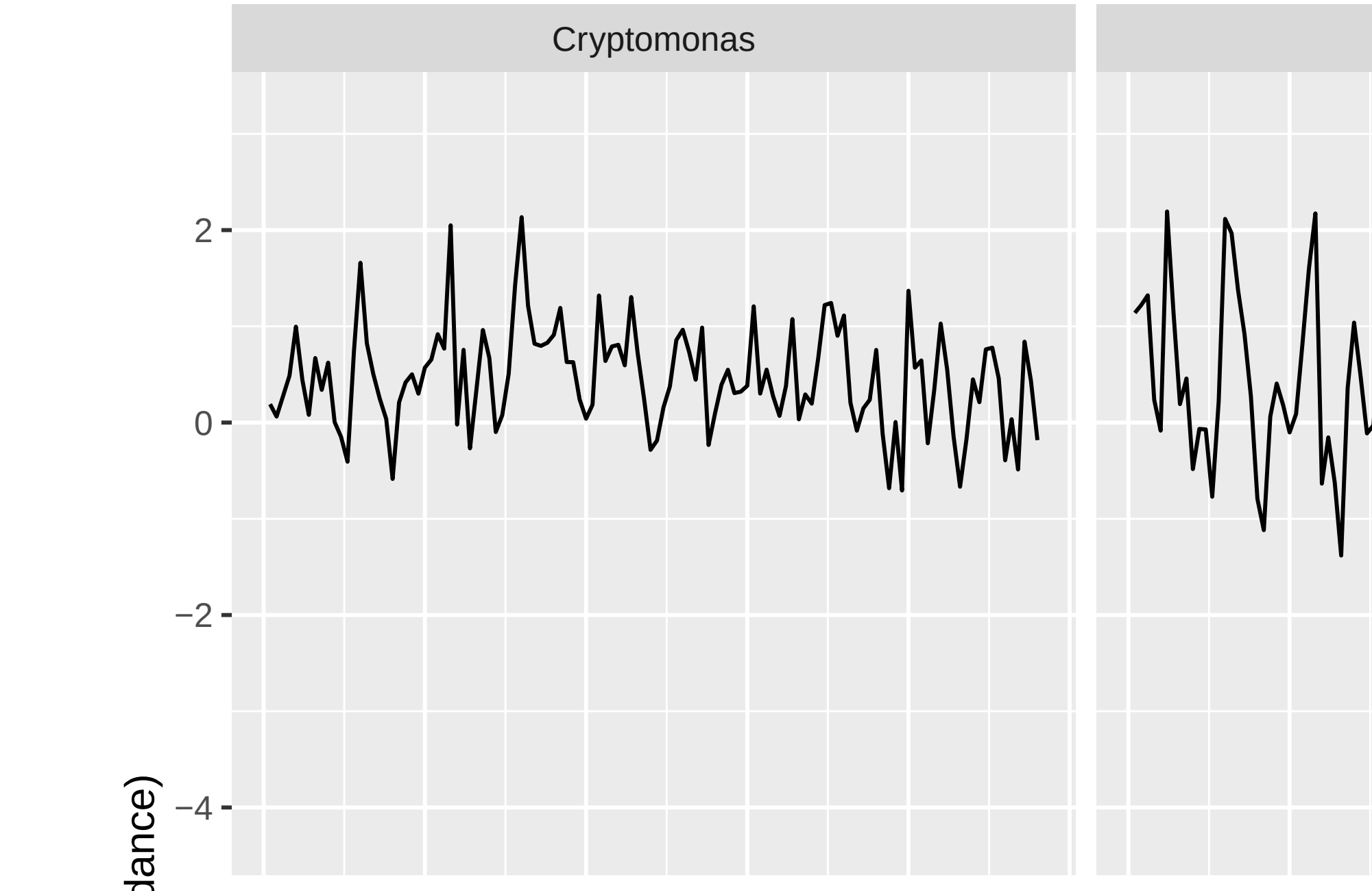
Dynamic Factor Analysis (DFA)

As an example of Bayesian DFA, we'll load the WA plankton dataset. There's 3 versions of the data, and we'll use the one that's been transformed. As a reminder,

```
+ 0s replaced with NAs
+ data have been z-scored
+ we only use data from 1980-1989 for simplicity
```

```
dat = lakeWAplanktonTrans
plankdat = dat[dat[,"Year"]>=1980 & dat[,"Year"]<1990,]
phytoplankton = c("Cryptomonas", "Diatoms", "Greens",
                  "Unicells", "Other.algae")
dat.spp.1980 = plankdat[,phytoplankton]
```

Dynamic Factor Analysis (DFA)



Running the model

For starters, we'll try a 3-trend model,

```
mod_3 = fit_dfa(y = t(dat.spp.1980), num_trends=3)
```

Trends need to be rotated (like MARSS)

Why? Identifiability

- Try re-ordering time series, and running DFA on each new dataset
- Results will be sensitive to time series order, BUT not after rotation
- Like MARSS we'll use varimax rotation

Use function we've written, rotate_trends

Attributes of rotated object

`Z_rot`, rotated Z matrix for each MCMC draw
`trends`, rotated trends for each MCMC draw
`Z_rot_mean`, mean Z across draws
`trends_mean`, mean trends across draws
`trends_lower`, lower 2.5% on trends
`trends_upper`, upper 97.5% on trends

Other variance structures

By default, the structure is `diagonal and equal`

Diagonal and unequal or shared variances can also be specified using `varIndx` argument. The diagonal and unequal structure would be called with

```
mod_3 = fit_dfa(y = t(dat.spp.1980), num_trends=3, varIndx
```

Uncertainty intervals on states

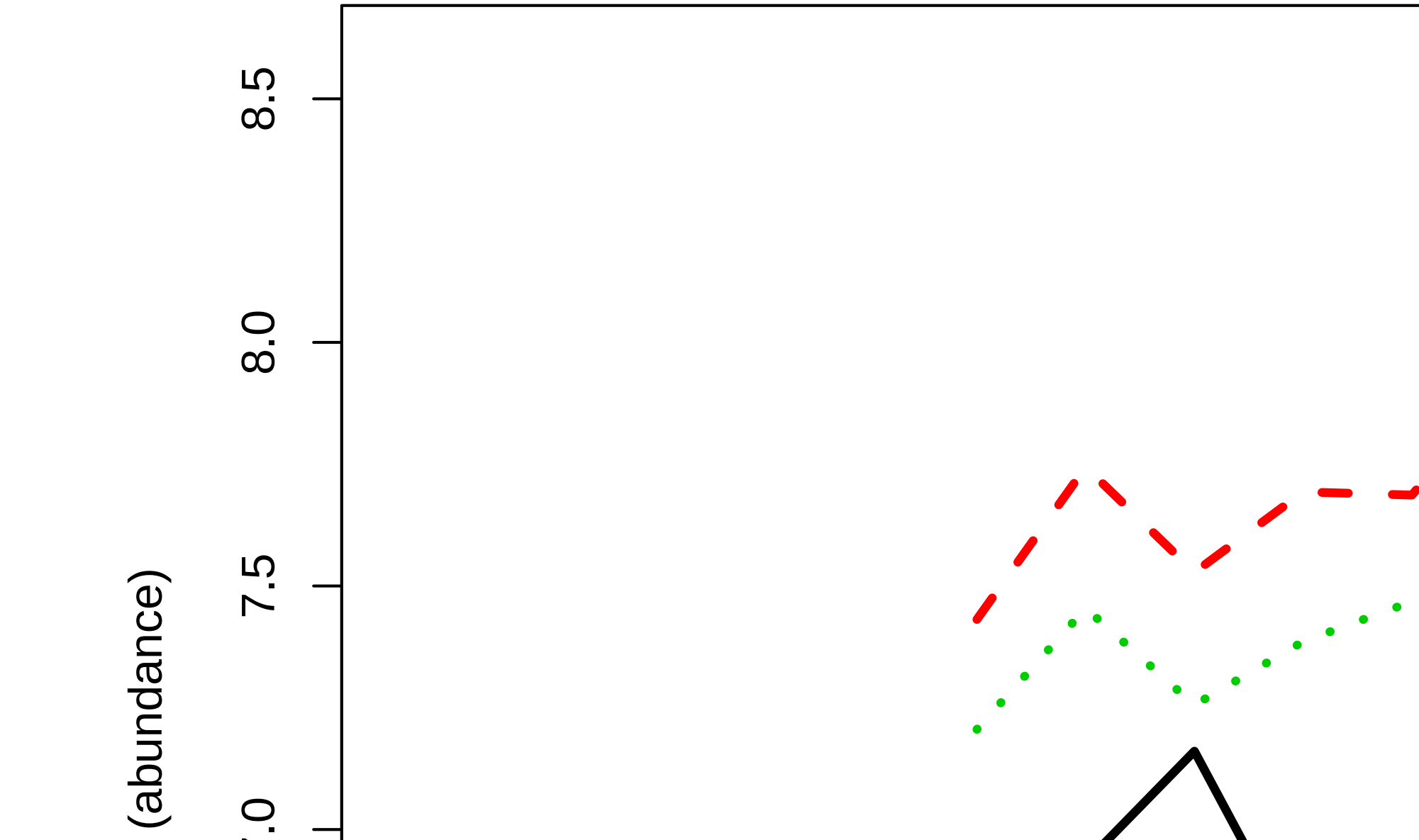
We often just present the trend estimates for DFA – but not uncertainty

Let's look at effect of missing data on DFA for the harbor seal dataset

```
data("harborSealWA")
```

Model

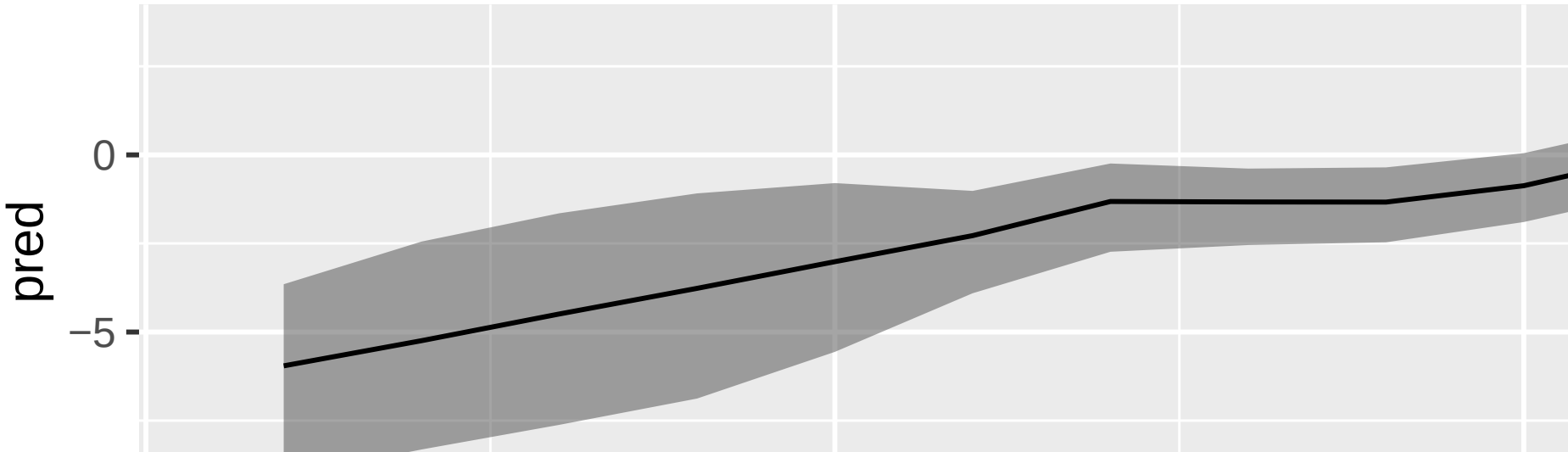
Assume single trend for the population



Extracting the predicted trend

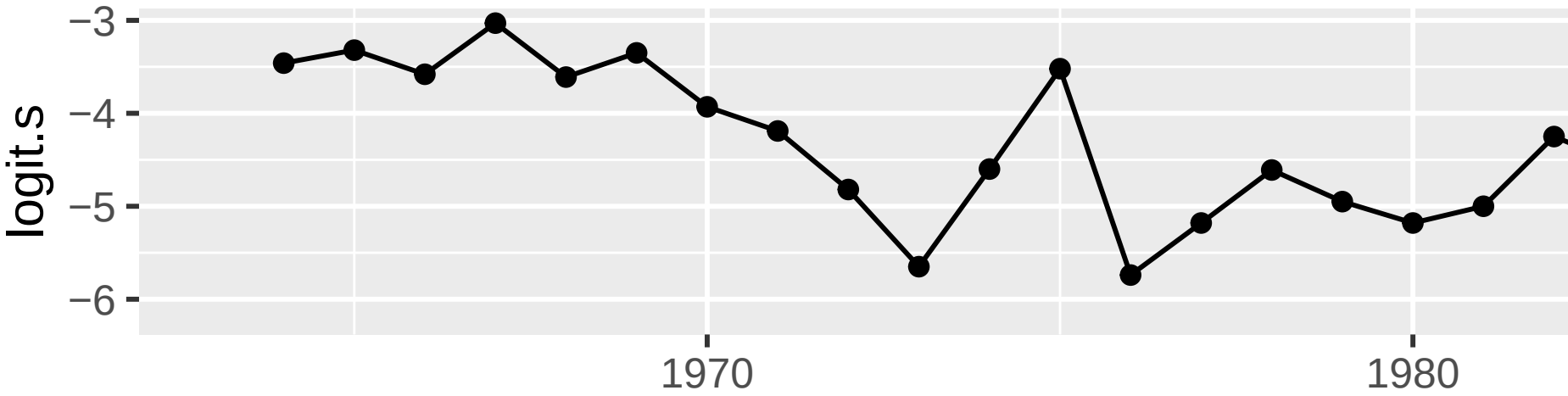
We'll extract predictions from the best model,

```
pars = extract(fit_dfa(y = t(harborSealWA[,-1]), num_trends
df = data.frame("time"=1:nrow(harborSealWA),
                "pred"=apply(pars$x[,1,], 2, mean),
                "low"=apply(pars$x[,1,], 2, quantile,0.025)
                "hi"=apply(pars$x[,1,], 2, quantile,0.975))
ggplot(df, aes(time, pred)) +
  geom_ribbon(aes(ymin=low,ymax=hi), fill="grey30", alpha=0
  geom_line()
```



Fitting a DLM with time varying intercept

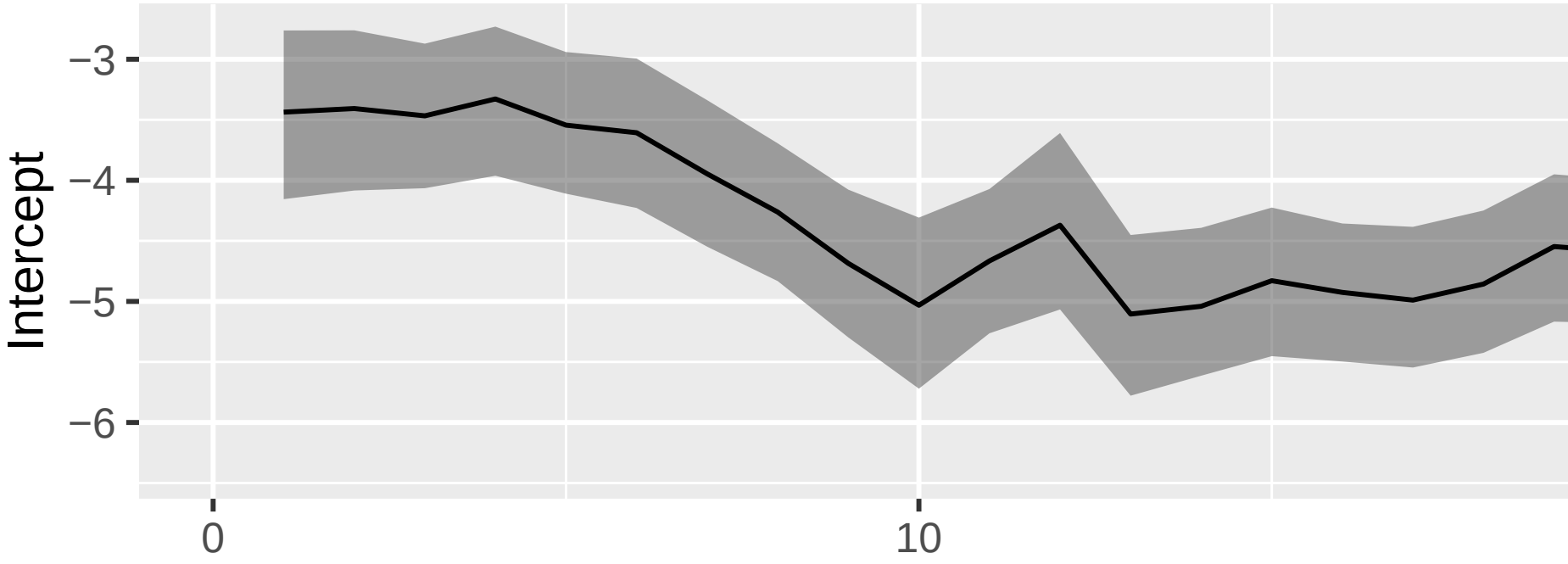
For comparison to MARSS, we'll use Mark's example of logit-transformed survival from the Columbia River. We can think about setting the DLM up in the slope or the intercept. For this first example, we'll do the latter.




```
mod = fit_stan(y = SalmonSurvCUI$logit.s,
               model_name="dlm-intercept")
```

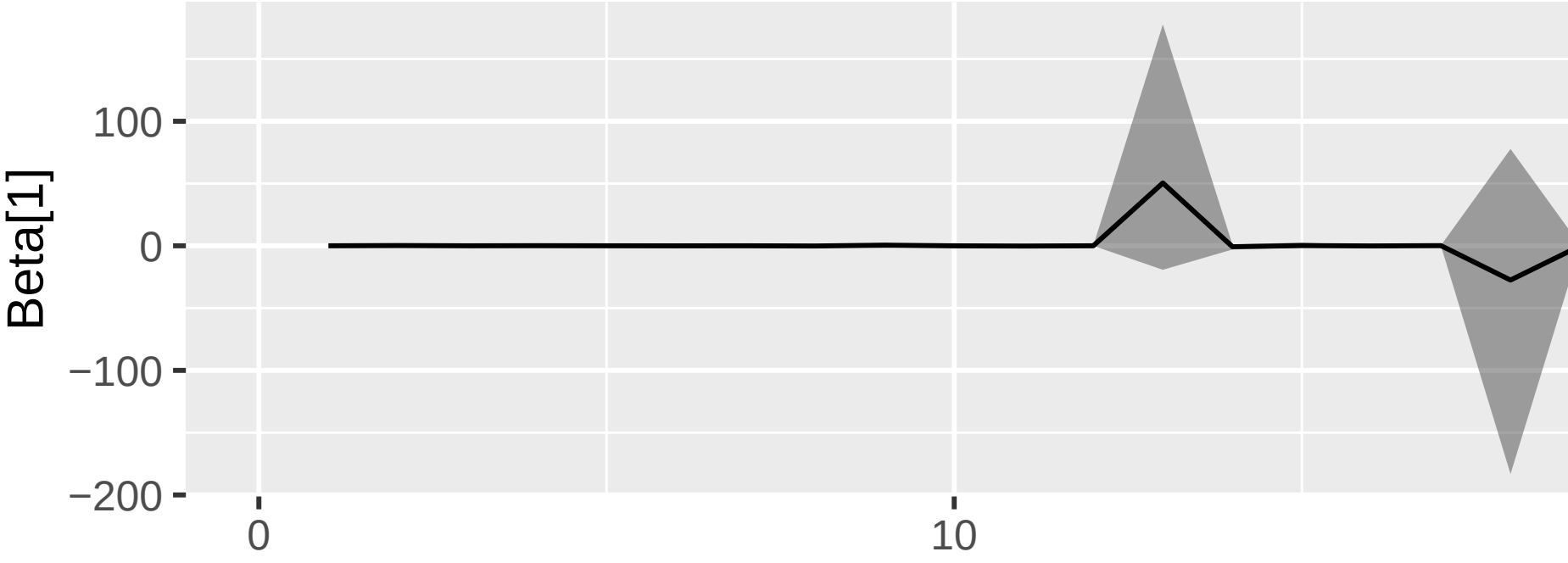
Fitting a DLM with time varying intercept

```
df = data.frame("year"=1:42,
"pred"=apply(extract(mod, "intercept", permuted=FALSE), 3,
"low"=apply(extract(mod, "intercept", permuted=FALSE), 3, q
"hi"=apply(extract(mod, "intercept", permuted=FALSE), 3, qu
ggplot(df, aes(year,pred)) +
  geom_ribbon(aes(ymin=low,ymax=hi),fill="grey30",alpha=0.5
  geom_line() + ylab("Intercept") + xlab("Time")
```



Fitting a DLM with single intercept and time-varying slope

```
mod = fit_stan(y = SalmonSurvCUI$logit.s,
x = SalmonSurvCUI$CUI.apr, model_name="dlm-slope")
```



Fitting a DLM time-varying intercept and time-varying slope

- Use model.matrix() to specify x

```
lmmod = lm(SalmonSurvCUI$logit.s ~ SalmonSurvCUI$CUI.apr)
x = model.matrix(lmmod)
```

```
lmmod = lm(SalmonSurvCUI$logit.s ~ SalmonSurvCUI$CUI.apr)
mod = fit_stan(y = SalmonSurvCUI$logit.s,
               x = model.matrix(lmmod),
               model_name="dlm"
```

Summary

- Bayesian implementation of time series models in Stan can do everything that MARSS can do and more!
- Very flexible language, great developer community
- Widely used by students in SAFS / UW / QERM / etc
- Please come to us with questions or modeling issues!