Applications of
Dynamic Linear Models

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FISH 507 – Applied Time Series Analysis

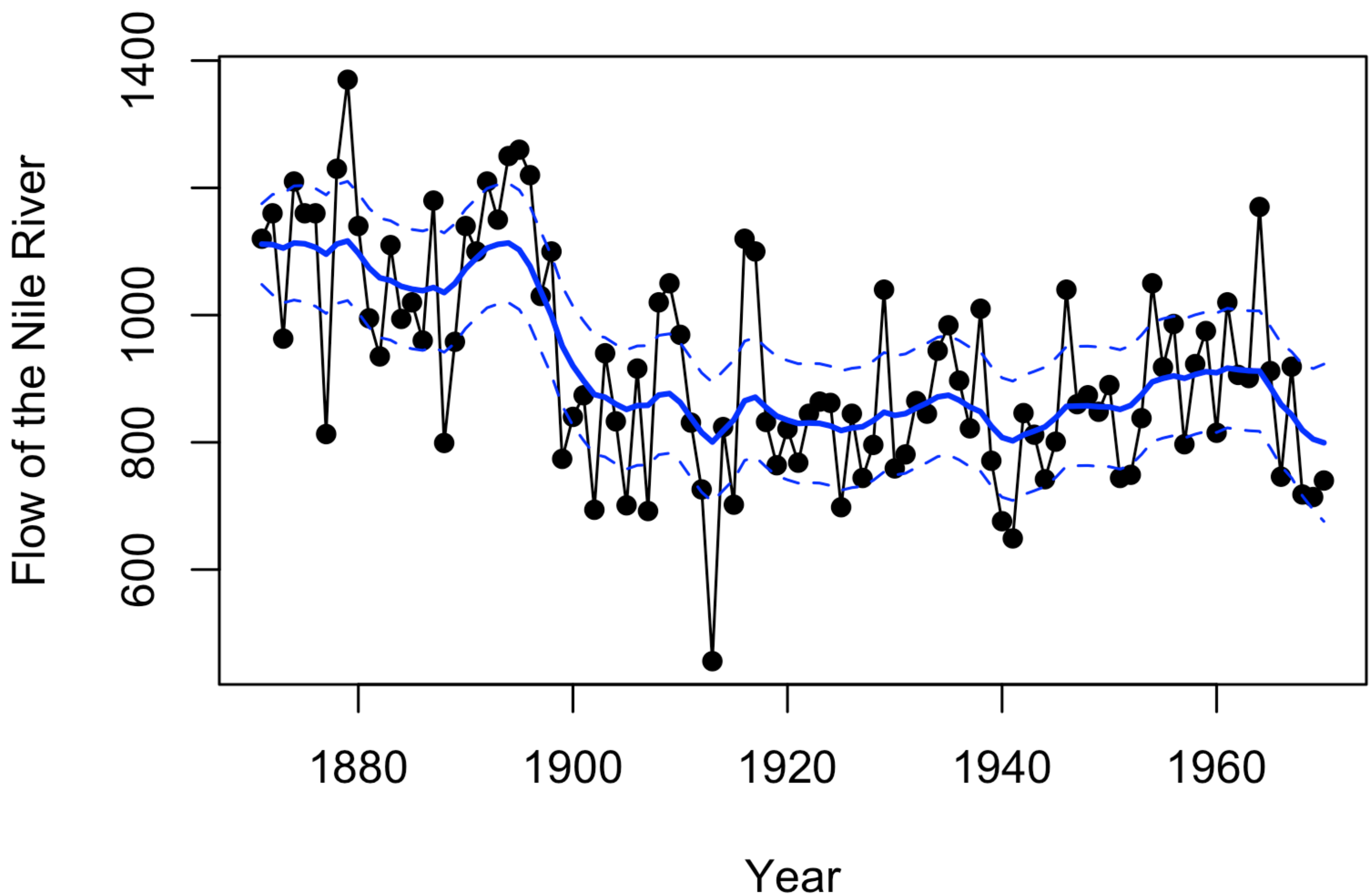
2 February 2017

Why use a DLM?

DLMs are useful if/when:

- The underlying level (intercept) changes over time (eg, flow of the Nile R)

DLM for changing level (intercept)

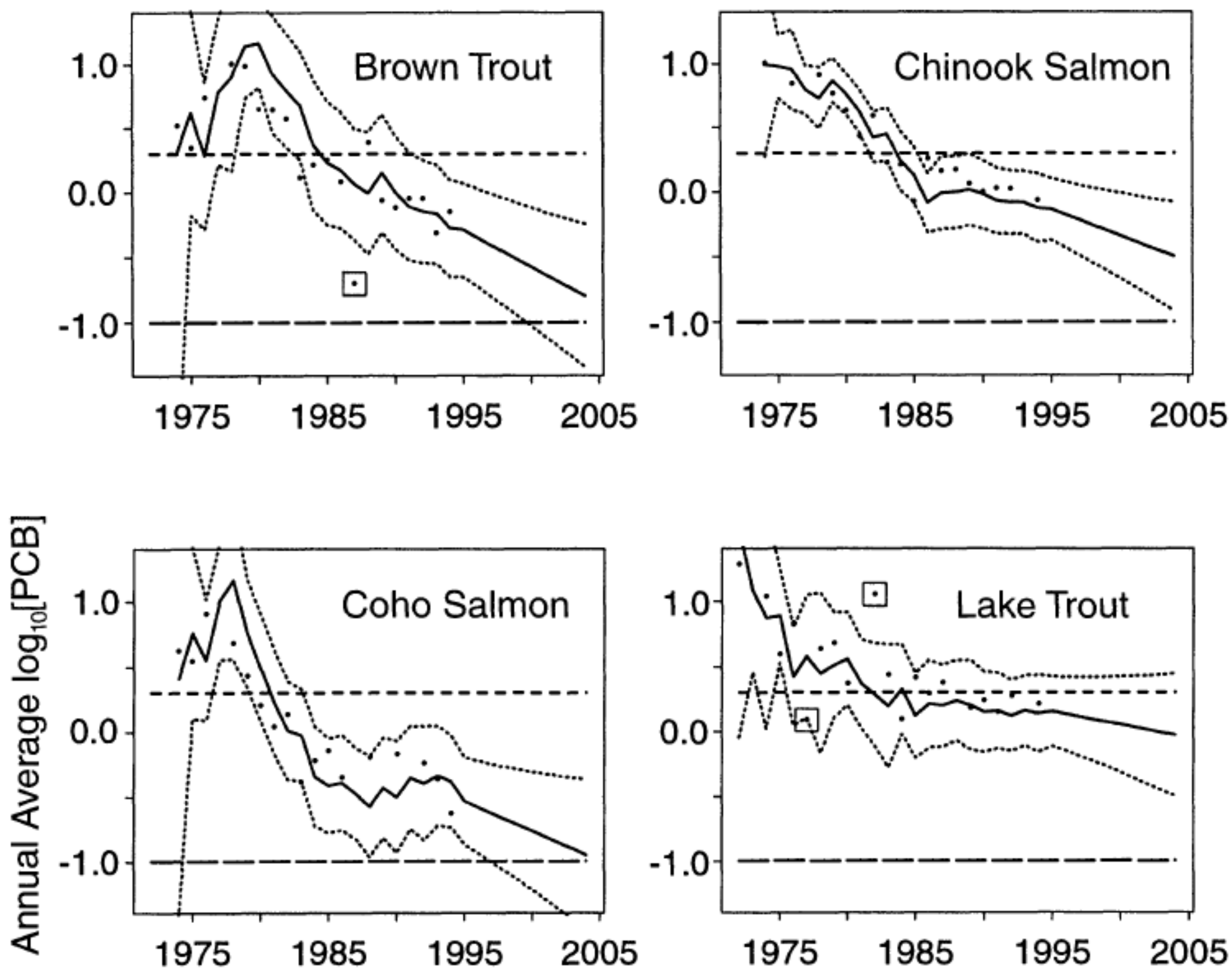


Why use a DLM?

DLMs are useful if/when:

- The underlying level (intercept) changes over time (eg, flow of the Nile R)
- The underlying growth (bias) changes over time (eg, PCB's in L Michigan salmonids)

DLM with changing growth (bias)



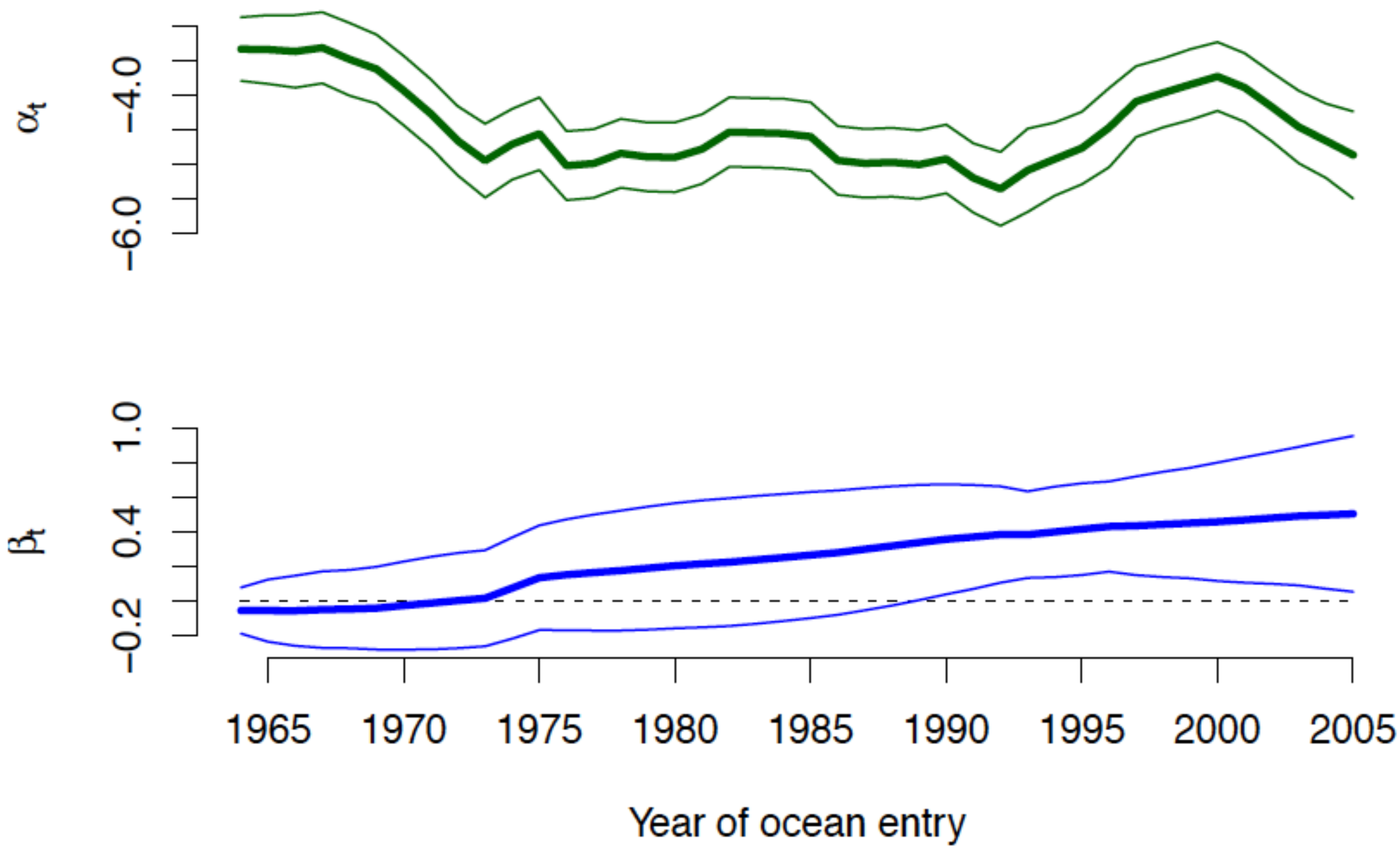
Lamon et al. (1998) *Ecol. Appl.*

Why use a DLM?

DLMs are useful if/when:

- The underlying level (intercept) changes over time (eg, flow of the Nile R)
- The underlying growth (bias) changes over time (eg, PCB's in L Michigan salmonids)
- The relationship between the response and predictor (slope) changes over time (eg, effect of upwelling on salmon survival)

DLM with changing effect (slope)



Scheuerell & Williams (2005) *Fish. Ocean.*

From finance to fisheries:
Using market models to evaluate returns versus risk for ESA-listed Pacific salmon

Mark Scheuerell
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Jennifer Griffiths
Stockholm University

Eli Holmes
NOAA Northwest Fisheries Science Center

Notions of risk in finance

- Financial markets are rife with various forms of risk
- For simplicity, let’s consider 2 broad categories:

1) Systematic (market) risk is vulnerability to large-scale events or outcomes that affect entire markets (eg, natural disasters, govt policy, terrorism)

2) Unsystematic (asset) risk is specific to particular securities or industries
(eg, droughts affect commodities like corn, but not oil; bad batteries affect Boeing, but not Microsoft)

Diversification

- By holding a diverse collection of assets (a portfolio), one can reduce unsystematic, but not systematic, risk
- Investing is inherently risky, but (rationale) investors are risk averse
- That is, if presented with 2 portfolios offering equal returns, they should choose the less risky one
- Thus, investors expect to be compensated with higher returns for accepting more risk & vice-versa

Estimating risks

- Portfolios can only reduce unsystematic risks, so one should understand the systematic risk of an asset before it is added to a portfolio
- Sharpe (1963) outlined a model whereby returns of various assets are related through a combination of a common underlying influence & random factors
- Total risk = Systematic risk + Unsystematic risk

The market model

- Many others (e.g., Treynor, Lintner, Beja) were also working on these ideas, which ultimately led to the "market model"

Returns of asset at time *t*

Returns of market at time *t*

$$r_{a,t} = \alpha_a + \beta_a r_{m,t} + v_{a,t}$$

Manager's skill

Correlated volatility (sensitivity to systematic risk)

Interpreting alpha

Value of α	Interpretation
$\alpha < 0$	Asset earns too little for its risk

*

*Expected value if market is "efficient" (*sensu* Fama 1970)

Interpreting beta

Value of β	Interpretation	Example
$\beta = 0$	Movement of asset is independent of market	Fixed-yield bond

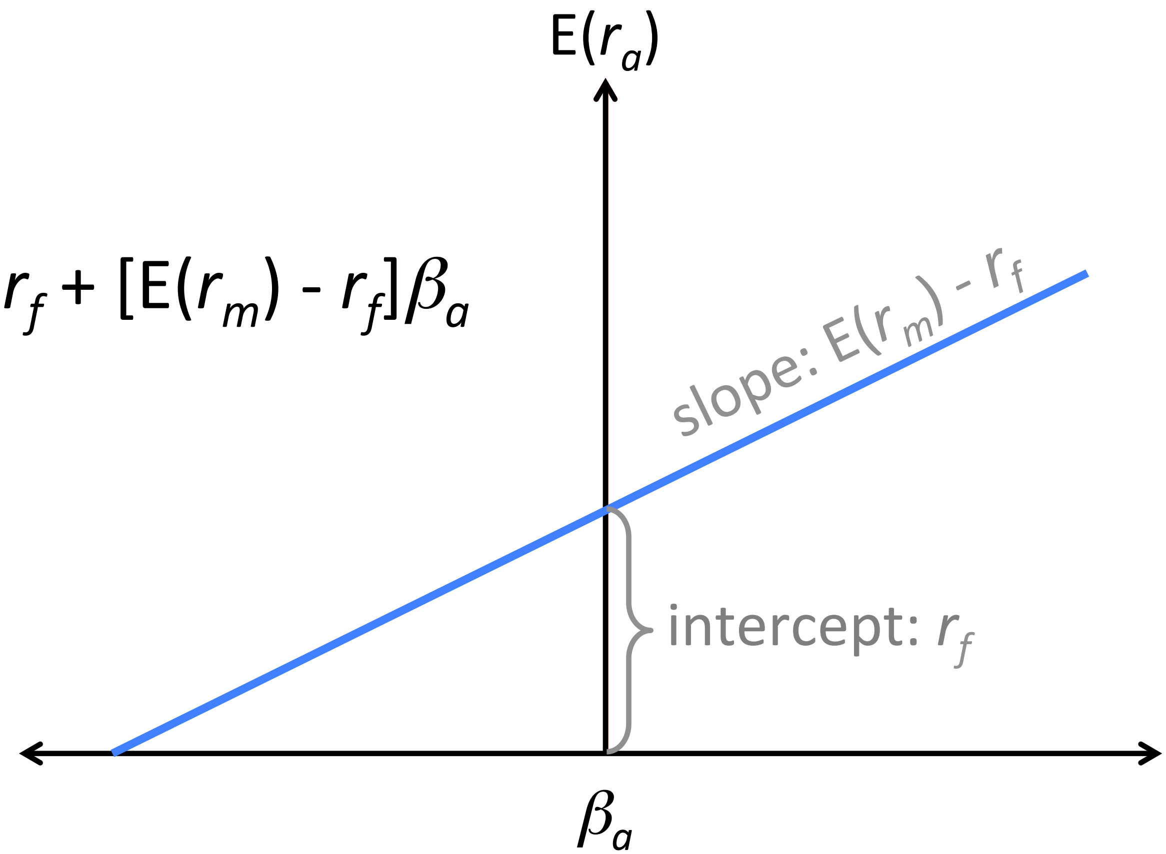
Capital Asset Pricing Model (CAPM)

- CAPM followed directly from the market model
- CAPM determines an expected rate of return necessary for an asset to be included in a portfolio based on:
  1) the asset's responsiveness to systematic risk (β);
  2) the expected return of the market; and
  3) the expected return of a risk-free asset (eg, US govt T-bills)
- CAPM is usually expressed via the security market line:

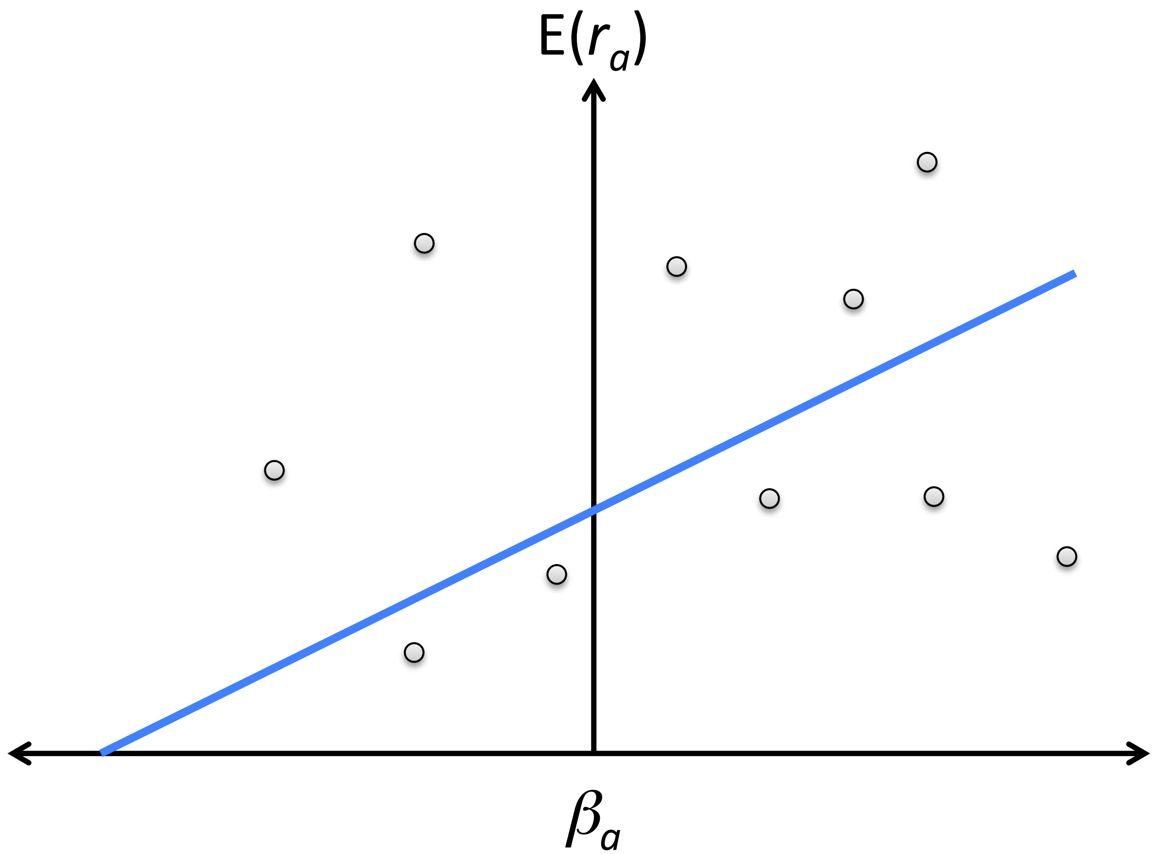
$$\mathrm{E}(r_a) = r_f + \left[\mathrm{E}(r_m) - r_f\right]\beta_a$$

Security market line (SML)

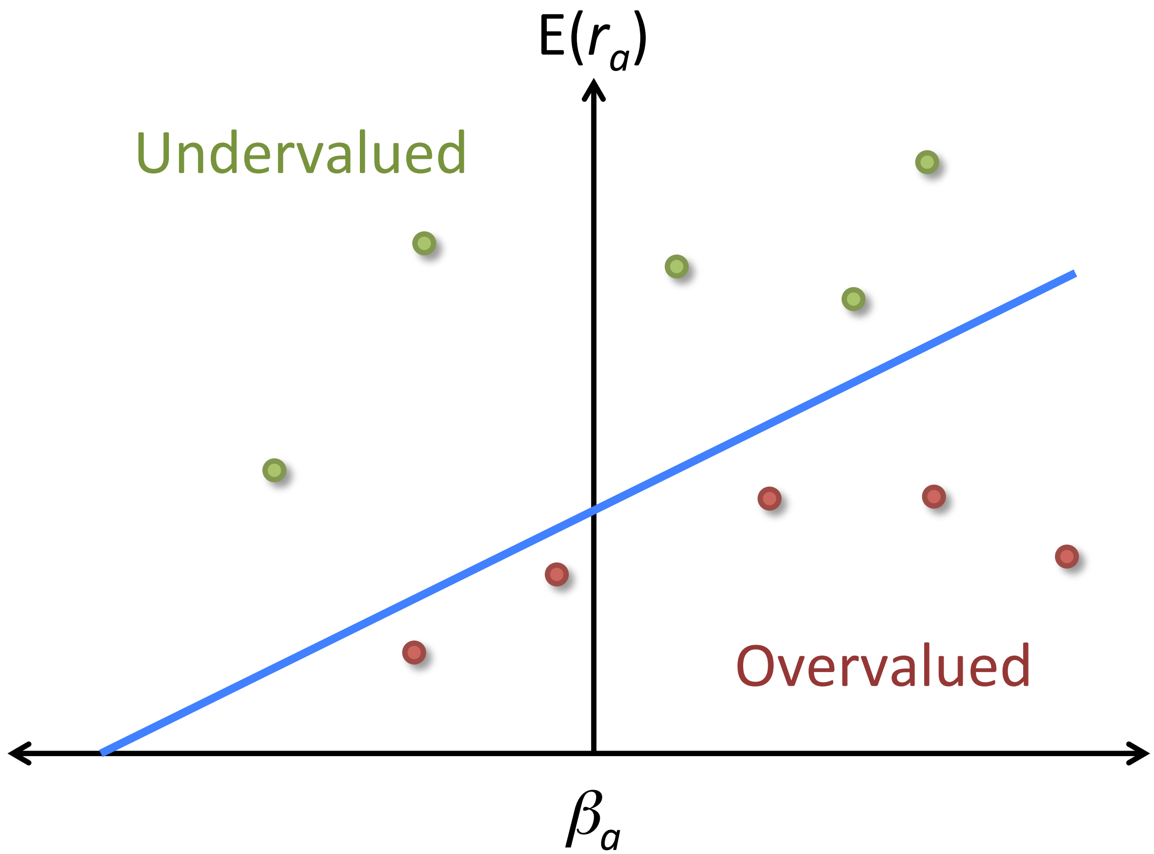
$$E(r_a) = r_f + [E(r_m) - r_f]\beta_a$$



Security market line

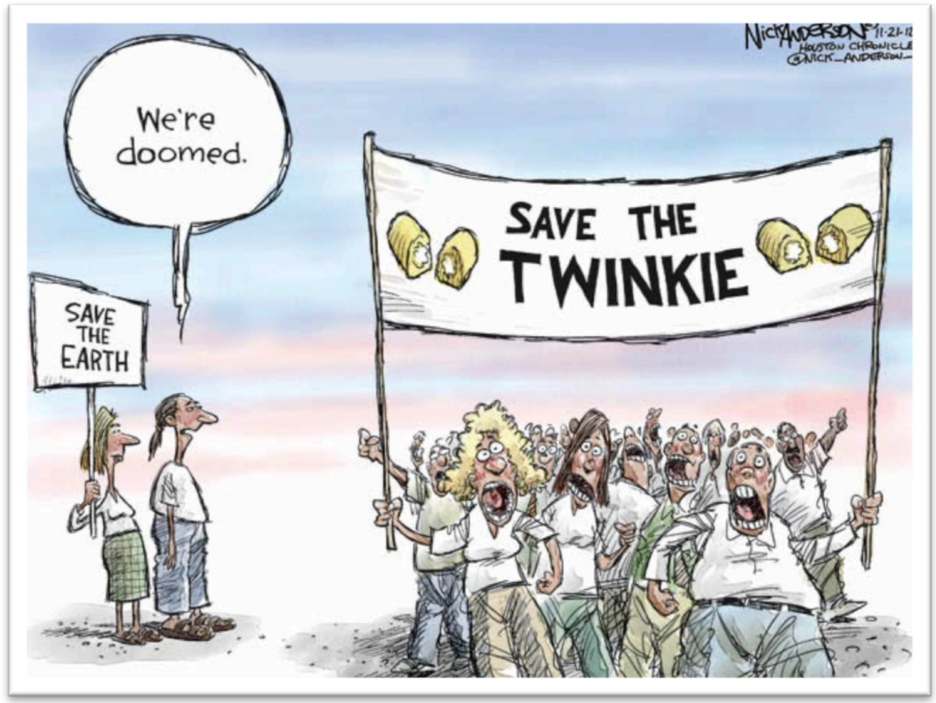


Security market line



Ecological analogues

<u>Many</u> ecologists study "risk" & "returns" in a conservation context



Portfolio ideas in nature

Vol 465 | 3 June 2010 | doi:10.1038/nature09060 nature

LETTERS

Population diversity and the portfolio effect in an exploited species

Daniel E. Schindler[1], Ray Hilborn[1], Brandon Chasco[1], Christopher P. Boatright[1], Thomas P. Quinn[1], Lauren A. Rogers[1] & Michael S. Webster[2]

LETTER

Synchronization and portfolio performance of threatened salmon

Jonathan W. Moore[1], Michelle McClure[1], Lauren A. Rogers[2], & Daniel E. Schindler[2]

[1] Northwest Fisheries Science Center, National Marine Fisheries Service, Seattle, WA 98110, USA
[2] School of Aquatic and Fishery Sciences, University of Washington, Seattle, WA 98195, USA

Journal of Animal Ecology

British Ecological Society

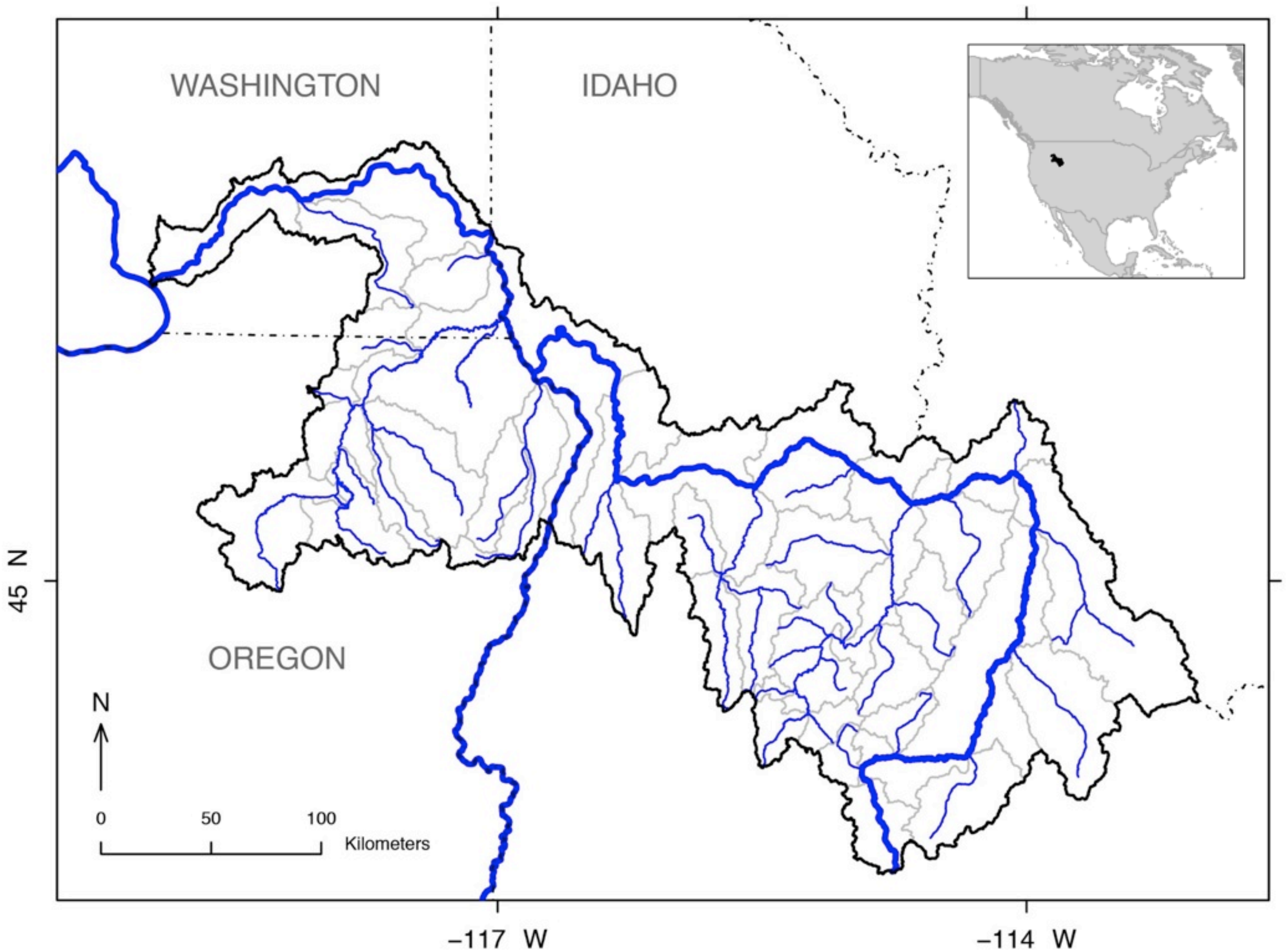
Journal of Animal Ecology 2014, **83**, 157–167 doi: 10.1111/1365-2656.12117

Spatial variation buffers temporal fluctuations in early juvenile survival for an endangered Pacific salmon

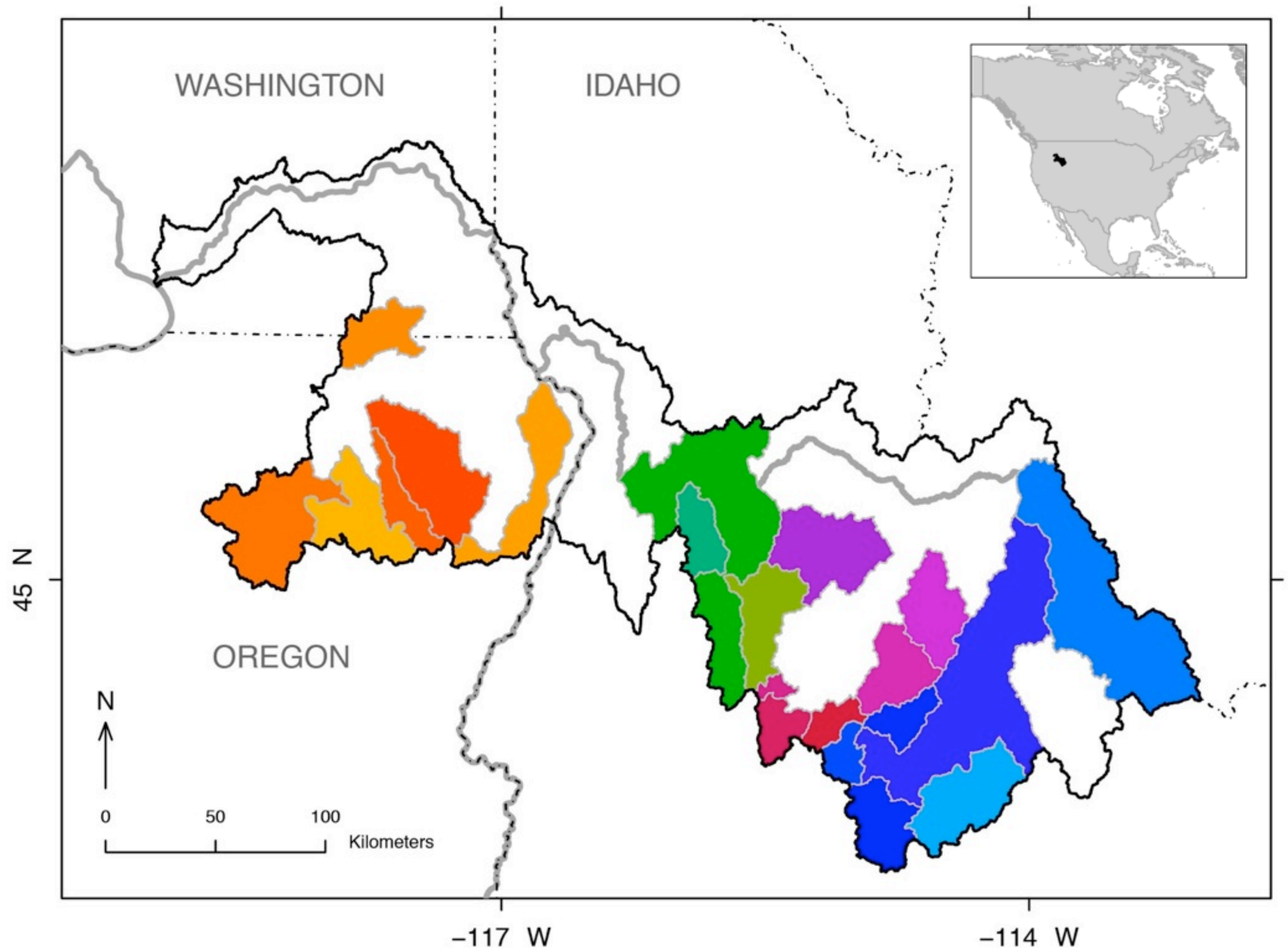
James T. Thorson[1]*, Mark D. Scheuerell[1], Eric R. Buhle[1] and Timothy Copeland[2]

[1]National Oceanic and Atmospheric Administration, National Marine Fisheries Service, Northwest Fisheries Science Center, 2725 Montlake Blvd. East, Seattle, WA 98112, USA; and [2]Idaho Department of Fish and Game 1414 East Locust Lane, Nampa, ID 83686, USA

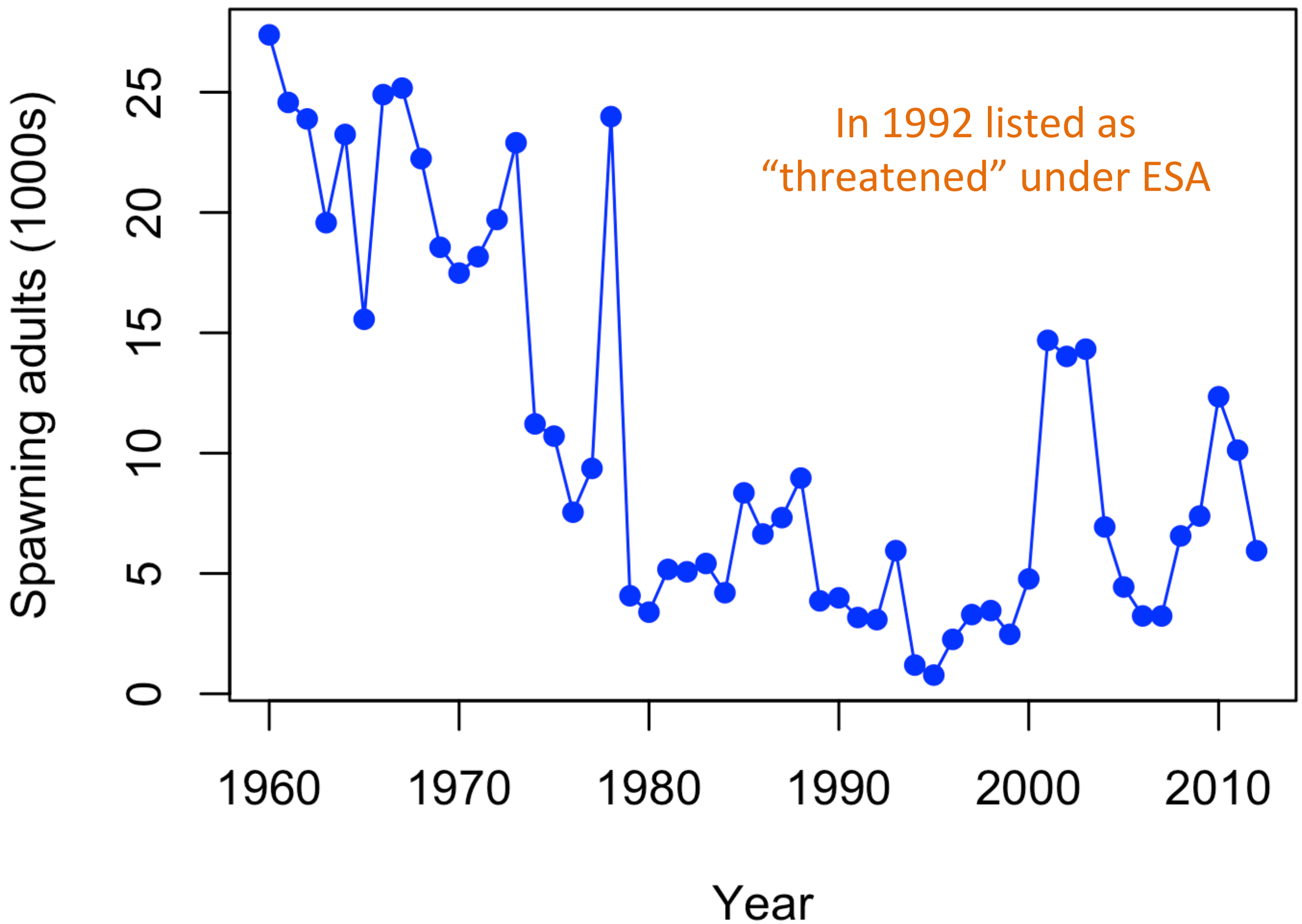
Snake R spring/summer Chinook



Snake R spring/summer Chinook

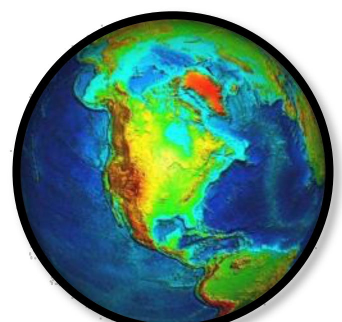


Time series of abundance



Reasons for decline

- ✓ Harvest
- ✓ Habitat degradation
- ✓ Hatchery operations
- ✓ Hydroelectric (& other) dams
- ✓ Climate
- ✓ Non-native species
- ✓ Marine-derived nutrients



What are the recovery options?

Recovery based on 4 “viable salmon” criteria:

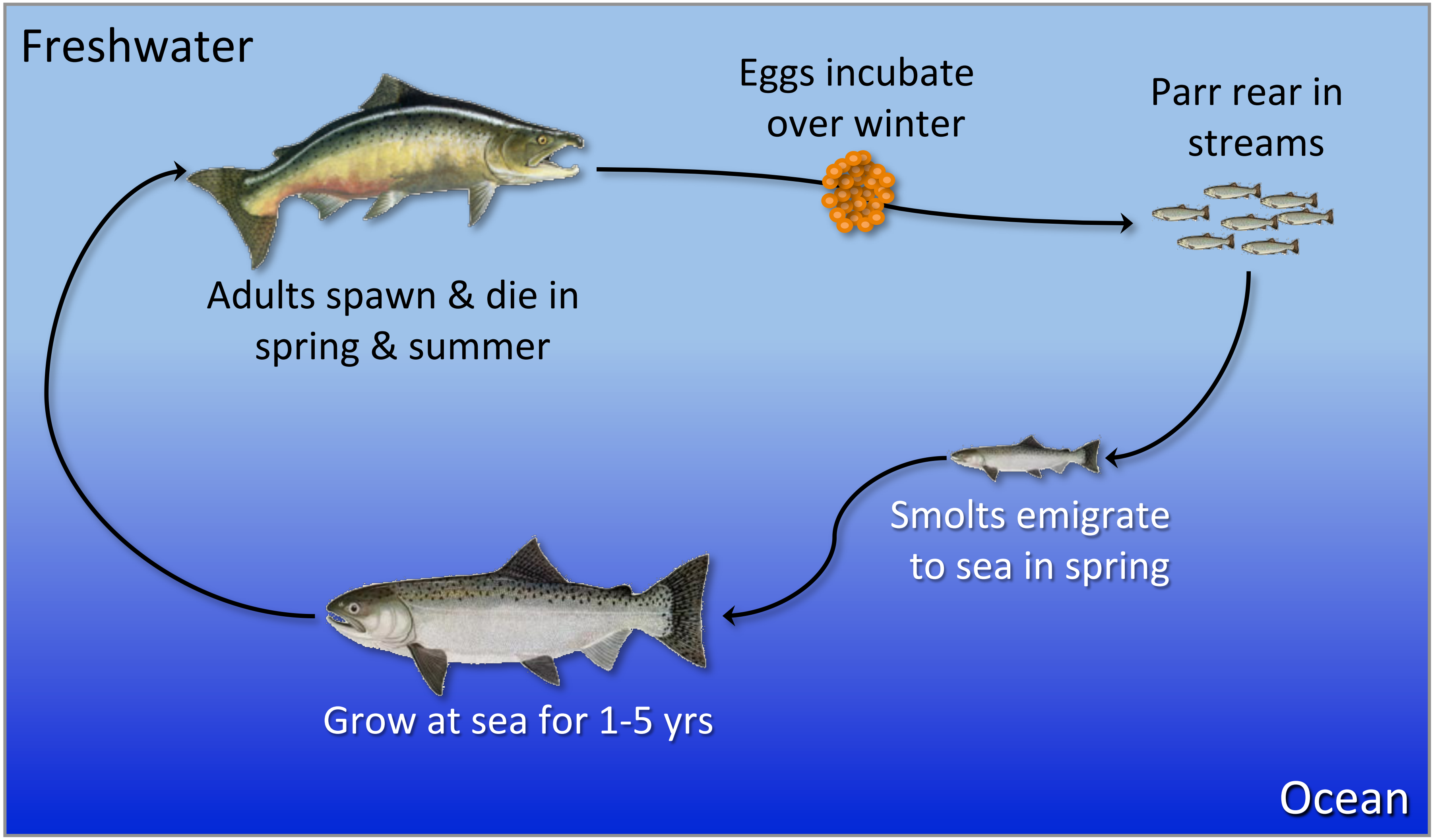
1. Productivity
2. Abundance
3. Spatial structure
4. Diversity

Asset, market & risk-free indices

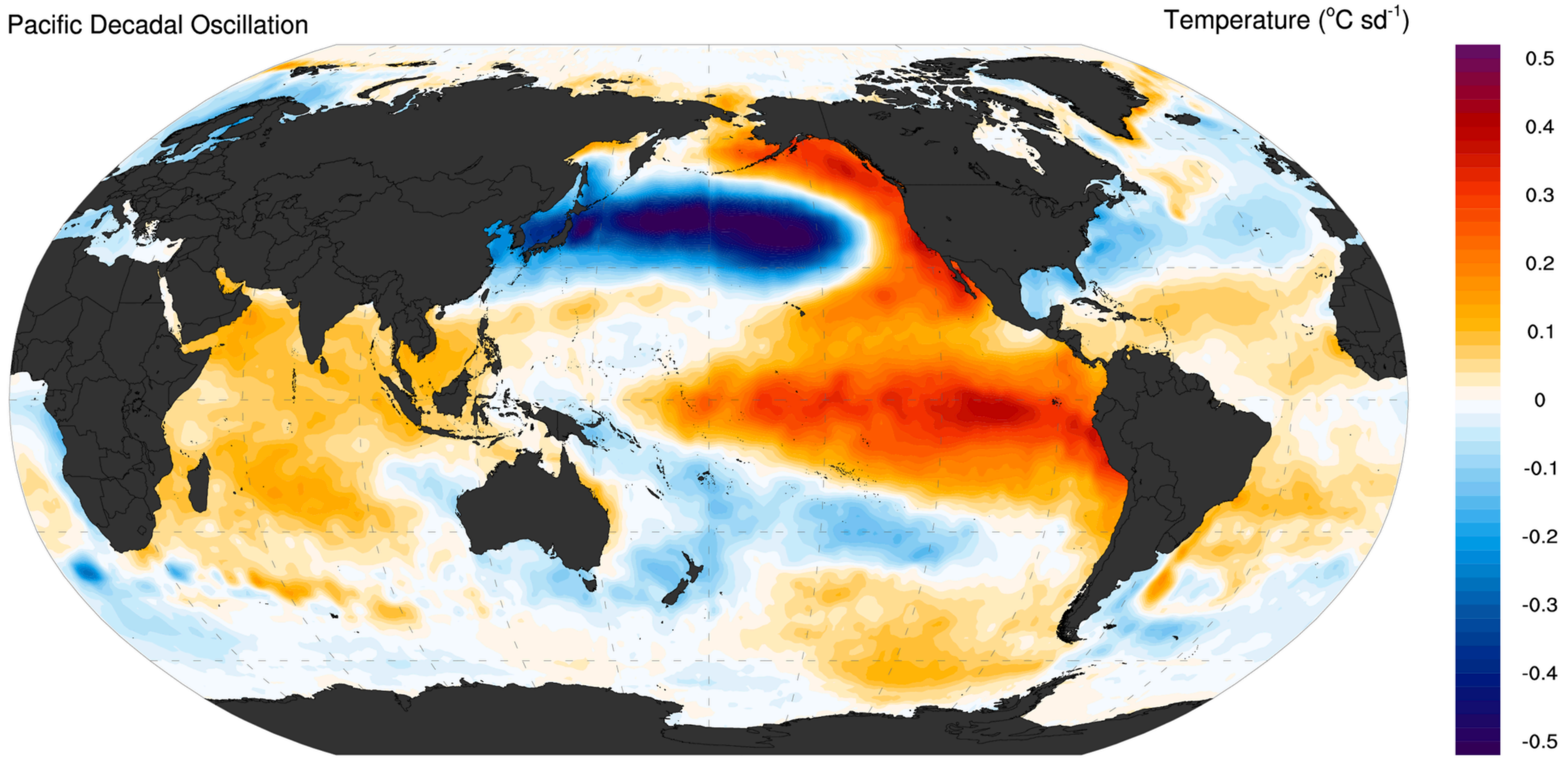
Assets

- ln[Recruits/Spawner] (productivity)
- ln[Spawners/ha] (abundance)

Salmon life cycle



The Pacific Decadal Oscillation



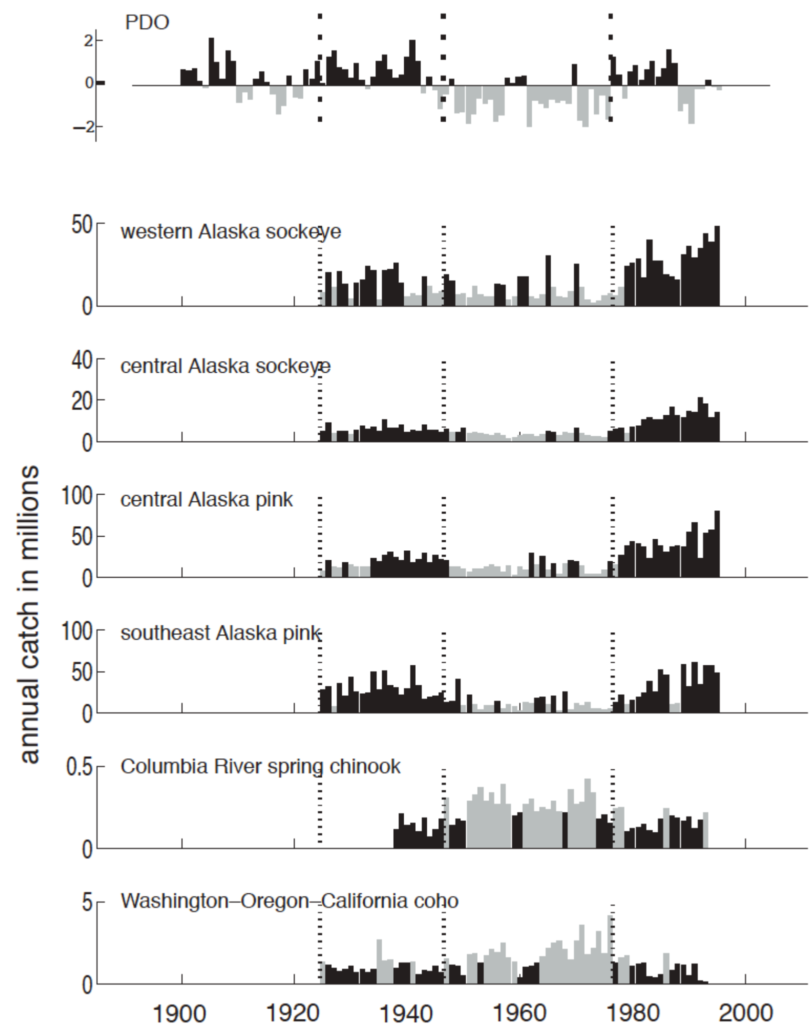
See Mantua et al. (1997) *Bull. Am. Meteor. Soc.*

The Pacific Decadal Oscillation

2 take-home messages:

1) Salmon production related to ocean conditions

2) The PDO is a pretty good indicator of lg-scale forcing



See Mantua et al. (1997) *Bull. Am. Meteor. Soc.*

Asset, market & risk-free indices

Assets

- ln[Recruits/Spawner] (productivity)
- ln[Spawners/ha] (abundance)

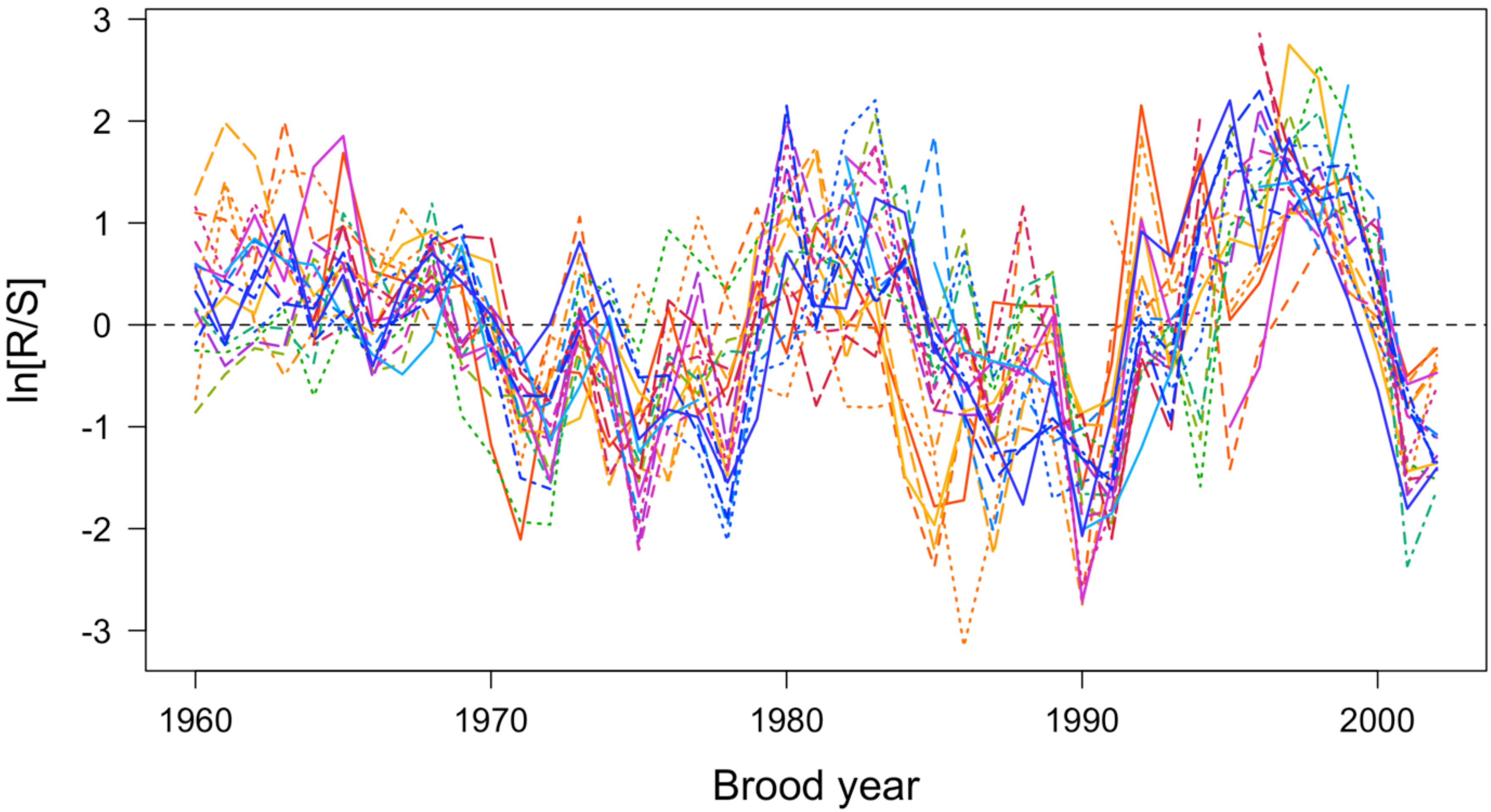
Market

- Pacific Decadal Oscillation (PDO) in brood yr + 2

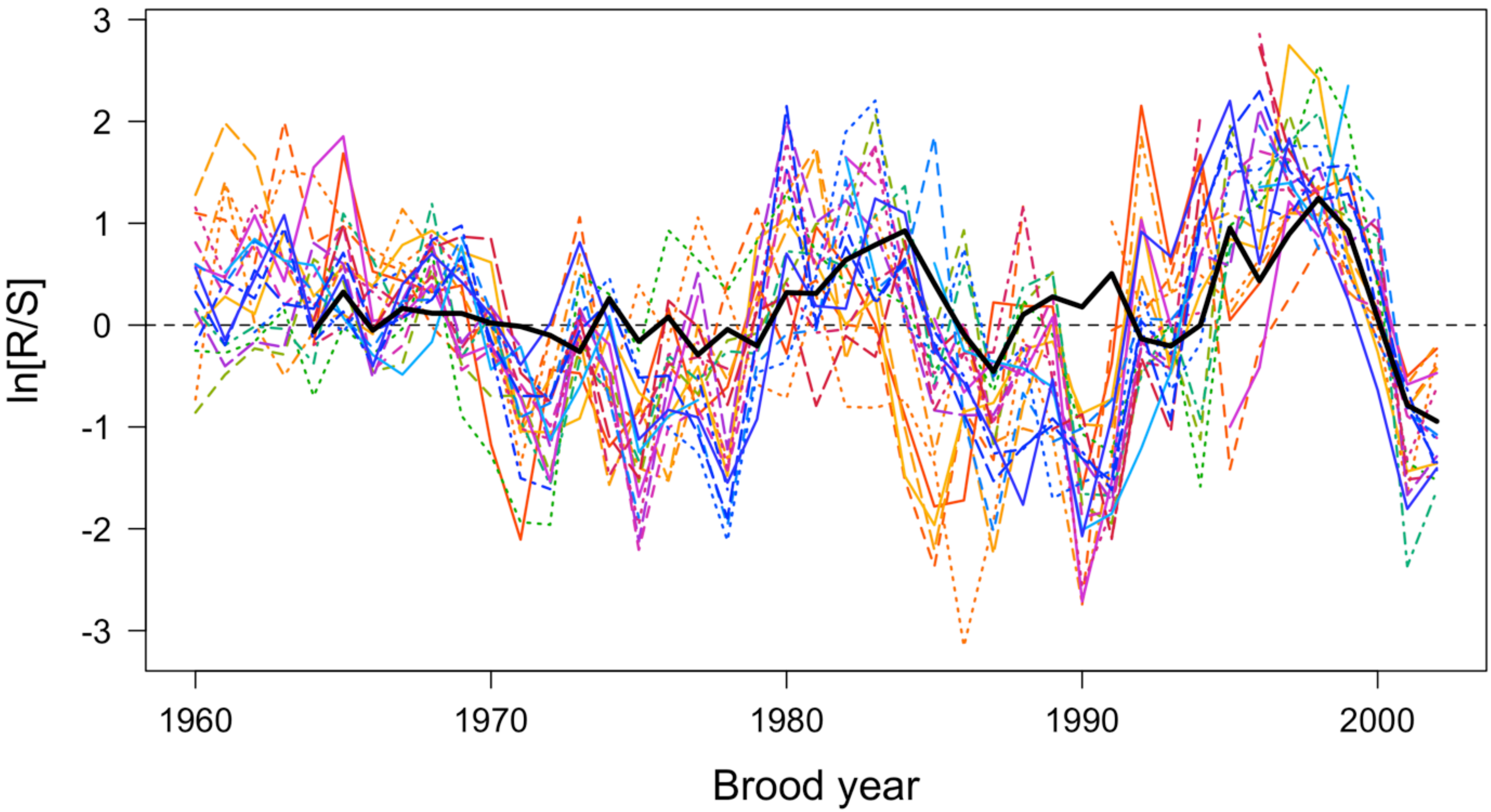
Risk-free

- Replacement (ln[R/S] = 0)
- ln[R/S] of John Day popn

Time series of “returns”



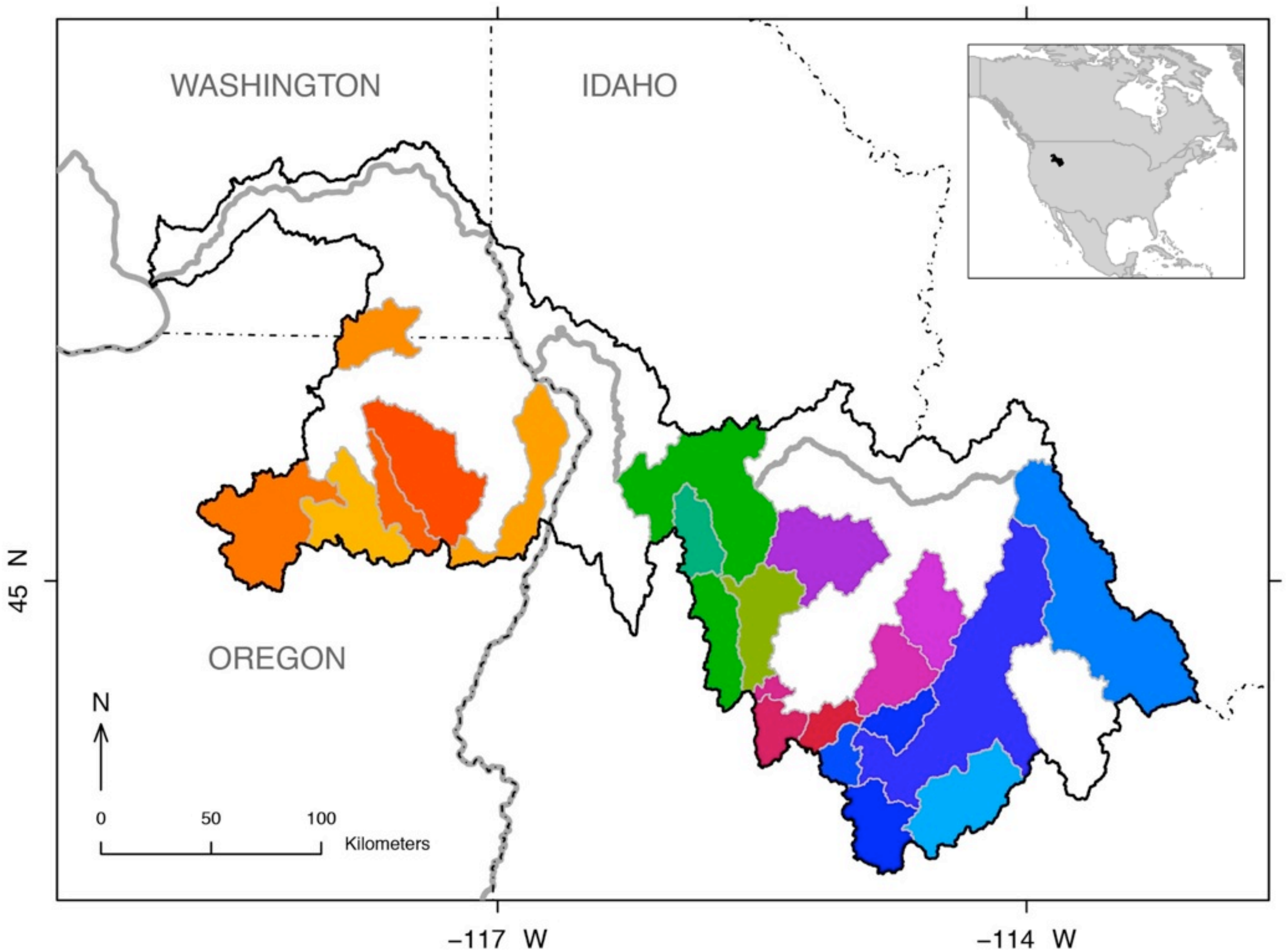
Time series of returns & risk-free



Fitting the market model

- In practice, the errors are often assumed to be Gaussian, and the model is solved via ordinary least squares
- This works well if the underlying relationship between asset & market is constant, but that rarely holds
- One option is to pass a moving window through the data, but window size affects accuracy & precision of β
- Better choice is to use a dynamic linear model (DLM)

Snake R spr/sum Chinook ESU



Multivariate DLM

- Here we will examine multiple assets at once, so we need a multivariate (response) DLM
- First, the obs eqn

$$r_{a,t} = \mathbf{R}_{m,t}\boldsymbol{\theta}_{a,t} + v_{a,t} \qquad v_t \sim \mathrm{N}(0,\sigma)$$

becomes

$$\mathbf{R}_t = \left(\mathbf{R}_{m,t} \otimes \mathbf{I}_n\right)\boldsymbol{\theta}_t + \mathbf{v}_t \qquad \mathbf{v}_t \sim \mathrm{MVN}(\mathbf{0},\boldsymbol{\Sigma})$$

Multivariate DLM – obs eqn

$$\mathbf{R}_t = \left(\mathbf{R}_{m,t} \otimes \mathbf{I}_n\right)\boldsymbol{\theta}_t + \mathbf{v}_t \qquad \mathbf{v}_t \sim \mathrm{MVN}(\mathbf{0}, \boldsymbol{\Sigma})$$

$$\begin{bmatrix} r_{1,t} \\ \vdots \\ r_{n,t} \end{bmatrix} = \begin{bmatrix} 1 & R_{m,t} \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Multivariate DLM – obs eqn

$$\mathbf{R}_t = \underline{(\mathbf{R}_{m,t} \otimes \mathbf{I}_n)\boldsymbol{\theta}_t} + \underline{\mathbf{v}_t} \qquad \mathbf{v}_t \sim \text{MVN}(\mathbf{0}, \boldsymbol{\Sigma})$$

$$\begin{bmatrix} r_{1,t} \\ \vdots \\ r_{n,t} \end{bmatrix} = \underline{\begin{bmatrix} 1 & 0 & 0 & r_{m,t} & 0 & 0 \\ 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & 0 & 1 & 0 & 0 & r_{m,t} \end{bmatrix}} \underline{\begin{bmatrix} \alpha_{1,t} \\ \vdots \\ \alpha_{n,t} \\ \beta_{1,t} \\ \vdots \\ \beta_{n,t} \end{bmatrix}} + \underline{\begin{bmatrix} v_{1,t} \\ \vdots \\ v_{n,t} \end{bmatrix}}$$

Multivariate DLM – evolution eqn

- The evolution eqn

$$\boldsymbol{\theta}_{a,t} = \mathbf{G}_t \boldsymbol{\theta}_{a,t-1} + \mathbf{w}_t \qquad \mathbf{w}_t \sim \mathrm{MVN}(\mathbf{0}, \mathbf{Q})$$

becomes

$$\boldsymbol{\theta}_t = (\mathbf{G}_t \otimes \mathbf{I}_n)\boldsymbol{\theta}_{t-1} + \mathbf{w}_t \qquad \mathbf{w}_t \sim \mathrm{MVN}(\mathbf{0}, \mathbf{Q})$$

$$\mathbf{G}_t = \mathbf{I}_2 \Rightarrow \mathbf{G}_t \otimes \mathbf{I}_n = \mathbf{I}_{2n}$$

$$\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \mathbf{w}_t$$

Multivariate DLM – evolution eqn

$$\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \mathbf{w}_t \qquad \mathbf{w}_t \sim \mathrm{MVN}(\mathbf{0}, \mathbf{Q})$$

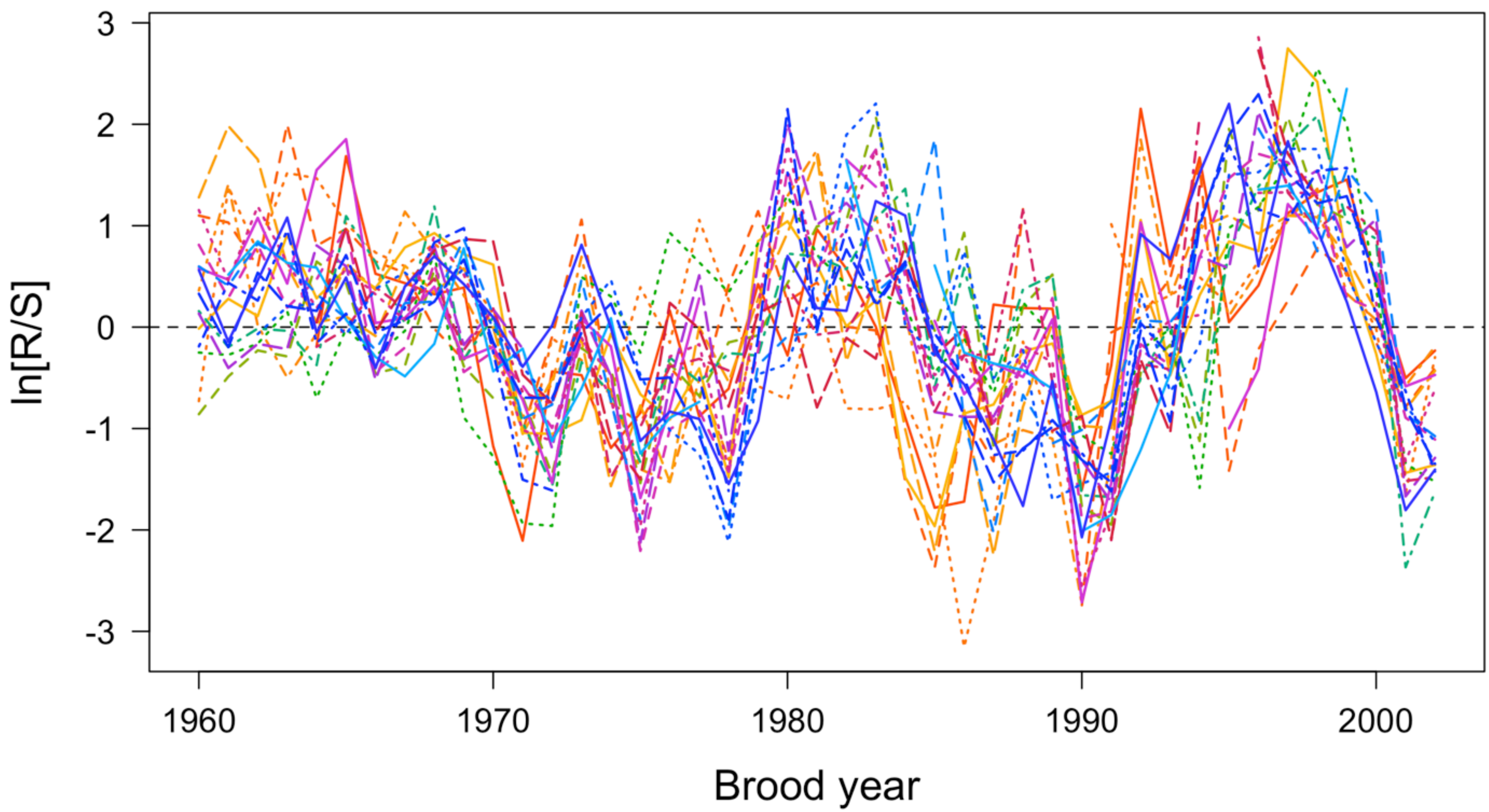
$$\begin{bmatrix} \alpha_{1,t} \\ \vdots \\ \alpha_{n,t} \\ \beta_{1,t} \\ \vdots \\ \beta_{n,t} \end{bmatrix} = \begin{bmatrix} \alpha_{1,t-1} \\ \vdots \\ \alpha_{n,t-1} \\ \beta_{1,t-1} \\ \vdots \\ \beta_{n,t-1} \end{bmatrix} + \begin{bmatrix} w_{1,t}^{(\alpha)} \\ \vdots \\ w_{n,t}^{(\alpha)} \\ w_{1,t}^{(\beta)} \\ \vdots \\ w_{n,t}^{(\beta)} \end{bmatrix}$$

Multivariate DLM – evolution eqn

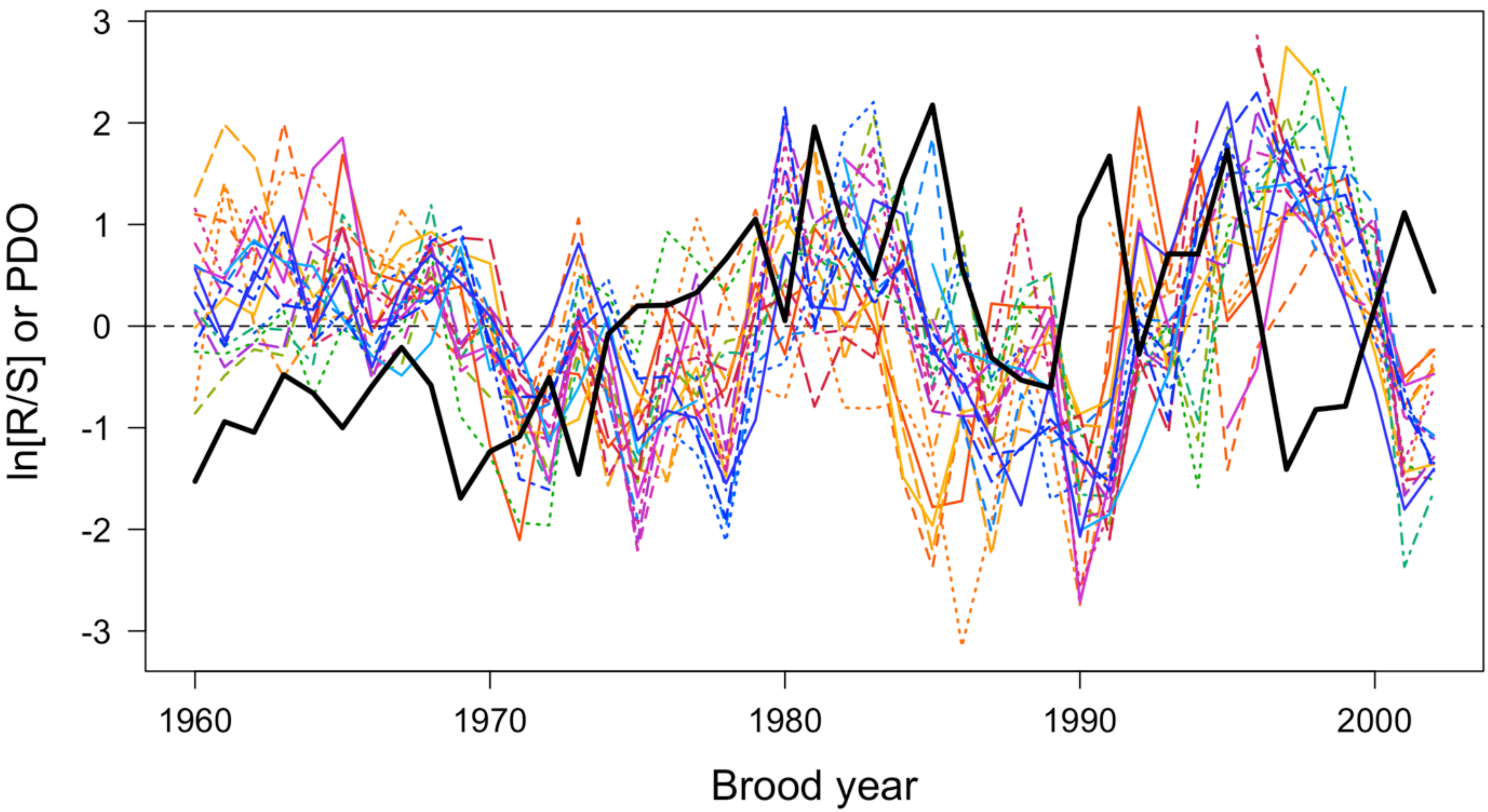
$$\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \mathbf{w}_t \qquad \mathbf{w}_t \sim \text{MVN}(\mathbf{0}, \underline{\mathbf{Q}})$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}^{(\alpha)} & \mathbf{0} \\ \mathbf{0} & \mathbf{Q}^{(\beta)} \end{bmatrix} \qquad \mathbf{Q}^{(\cdot)} = \begin{bmatrix} q_1^{(\cdot)} & c_{21}^{(\cdot)} & \cdots & c_{n1}^{(\cdot)} \\ c_{12}^{(\cdot)} & q_2^{(\cdot)} & & c_{n2}^{(\cdot)} \\ \vdots & & \ddots & \vdots \\ c_{1n}^{(\cdot)} & c_{2n}^{(\cdot)} & \cdots & q_n^{(\cdot)} \end{bmatrix}$$

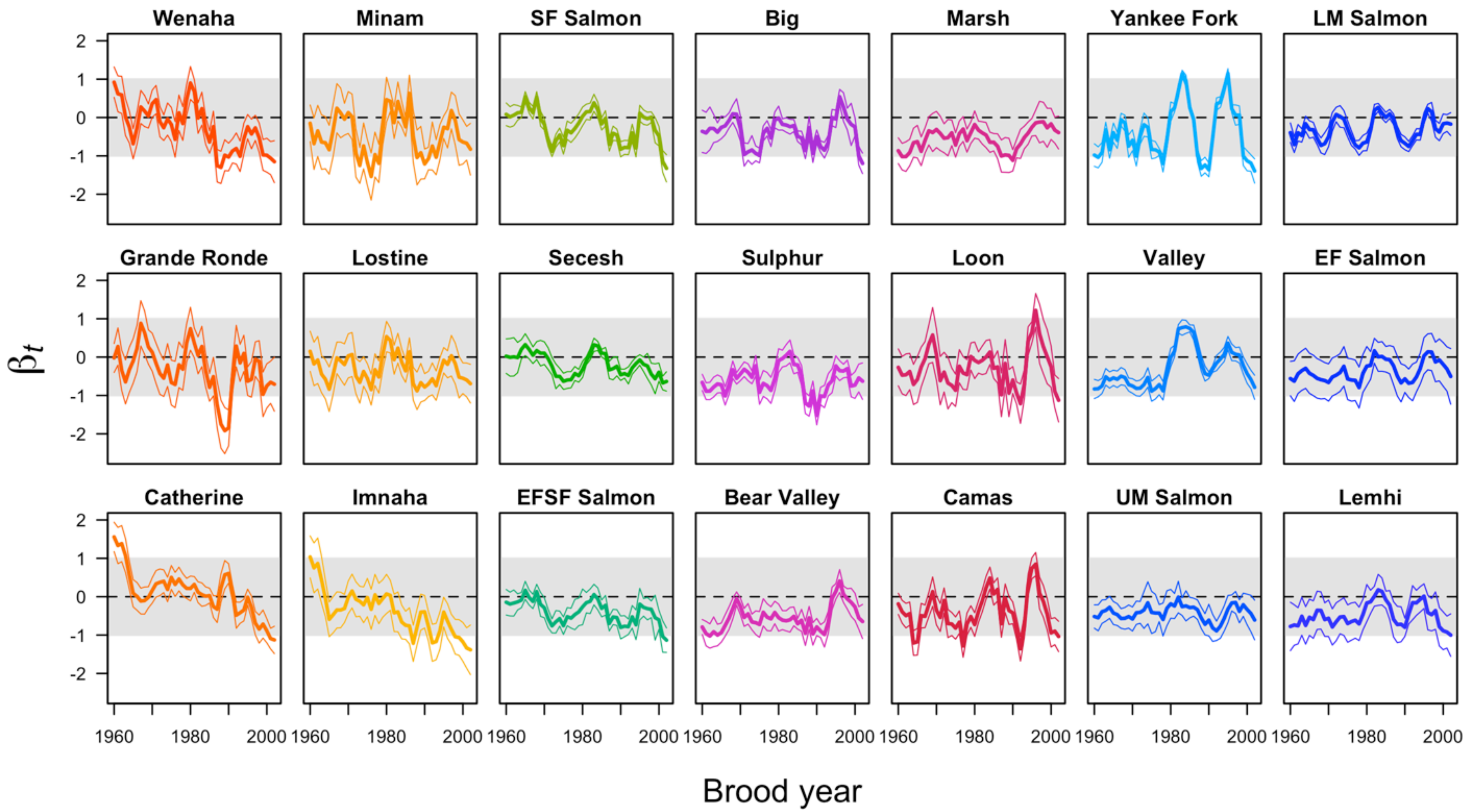
Time series of "returns"



Time series of "returns" & PDO

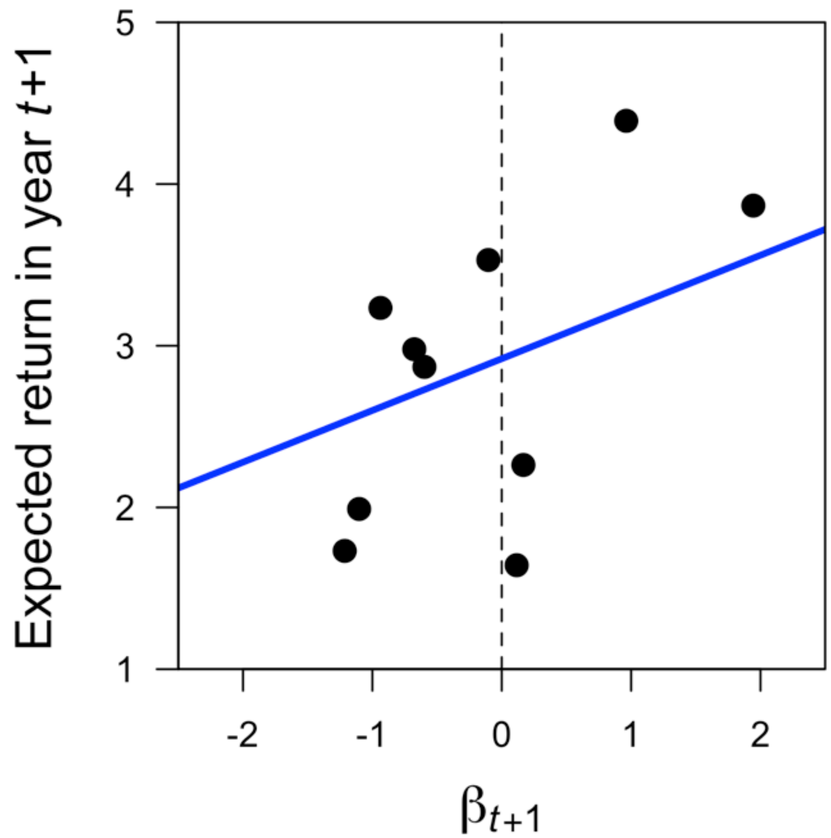


Results – time series of betas

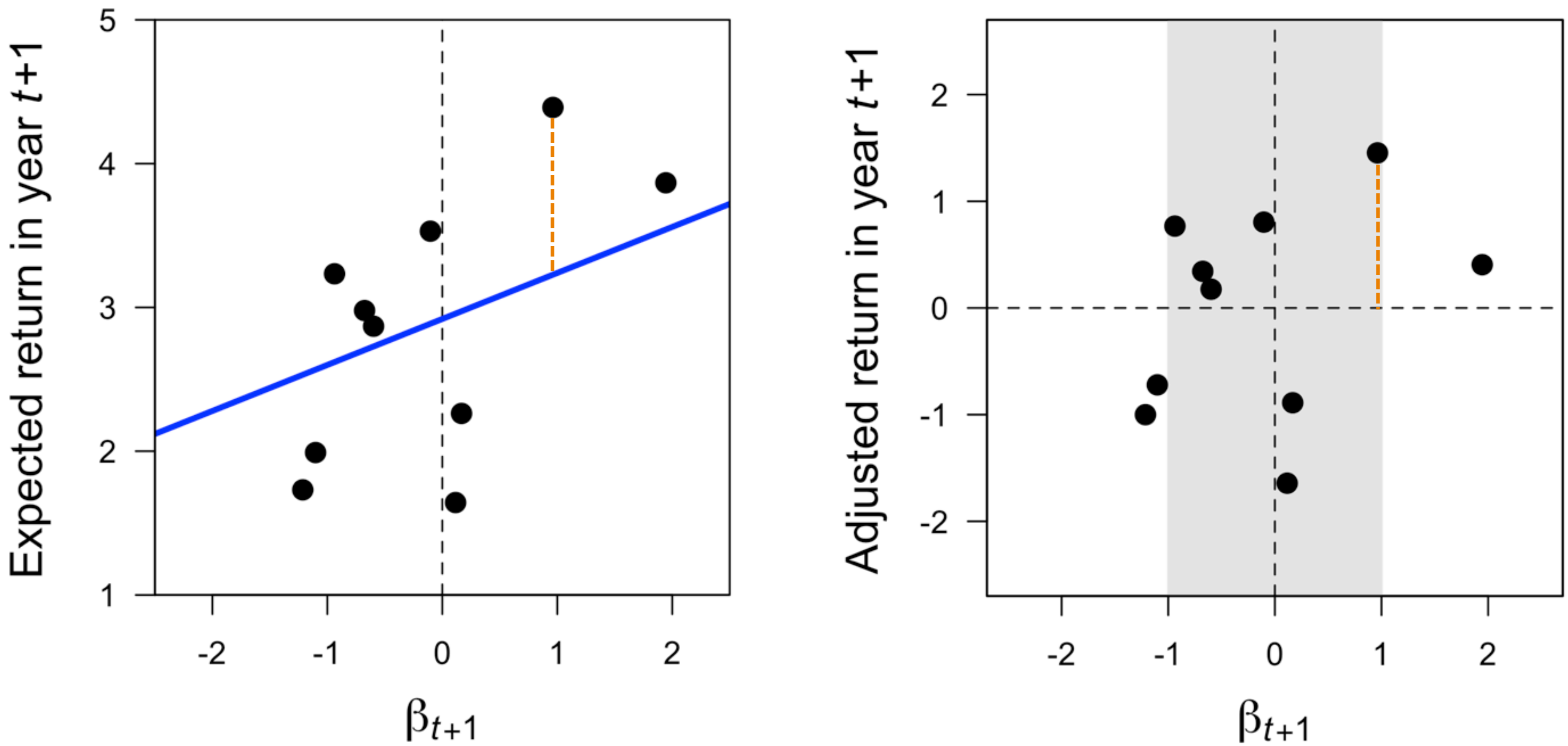


Market = PDO

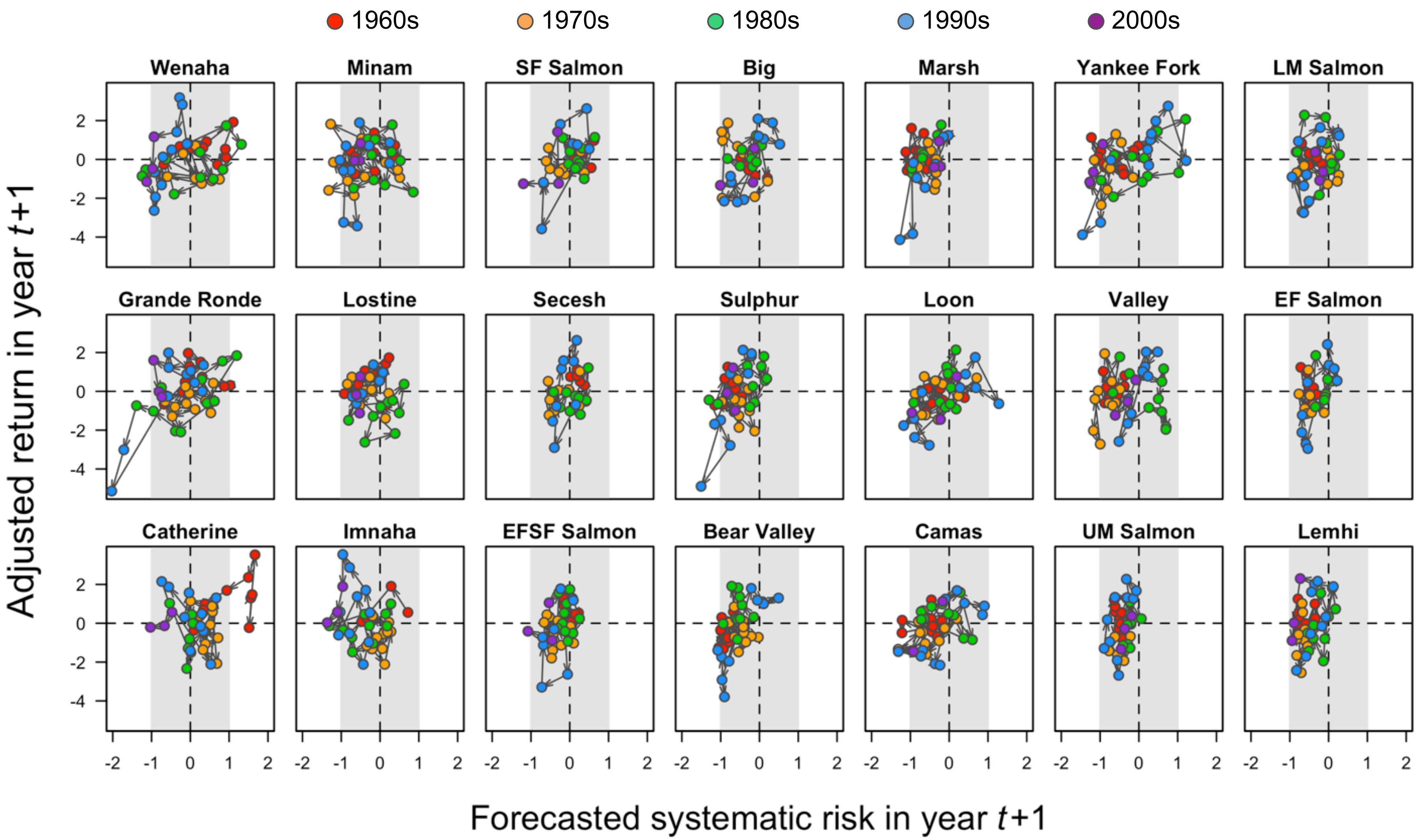
Security market line



Security market line

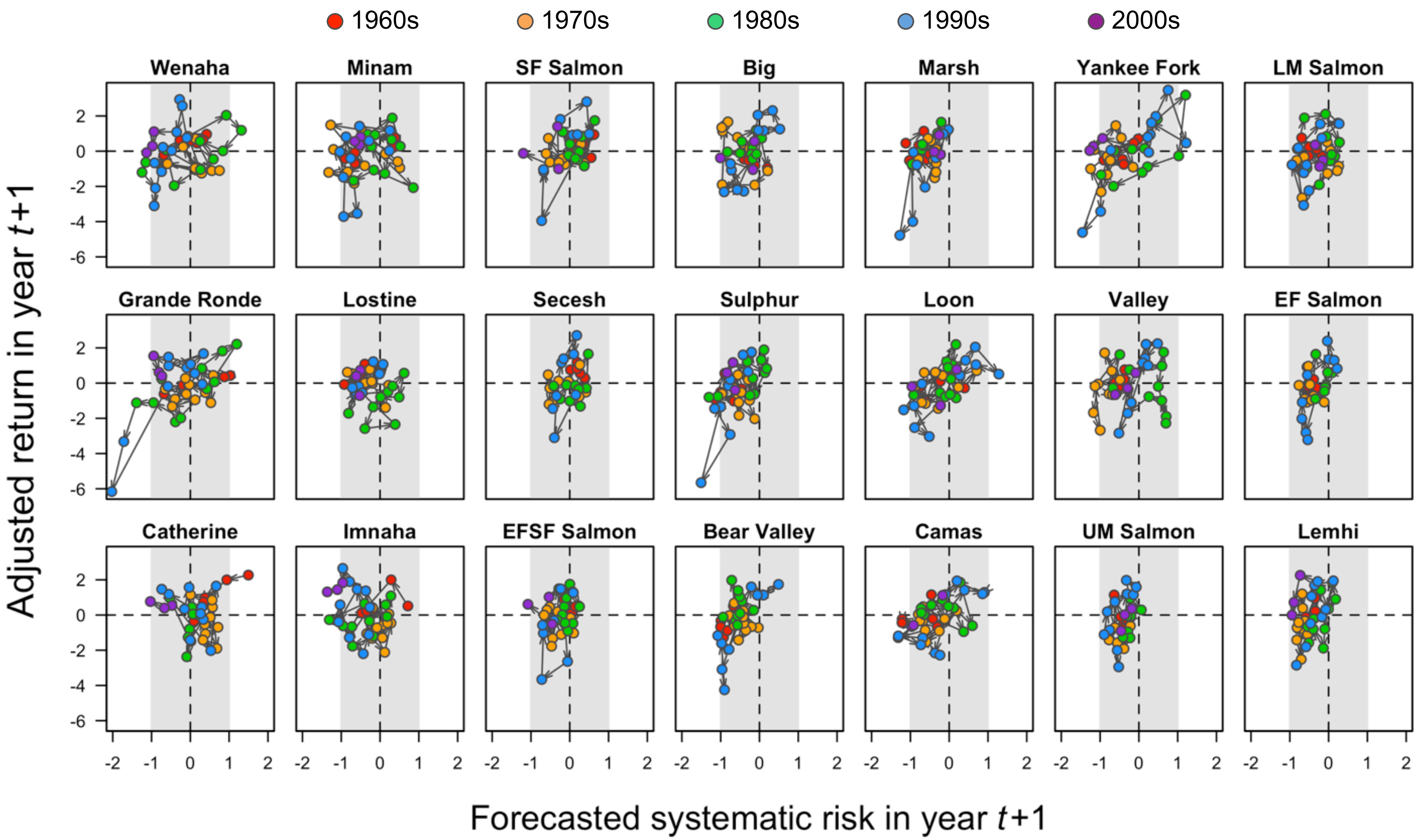


Results – CAPM



Market = PDO; *Risk-free* = 0

Results – CAPM



Market = PDO; *Risk-free* = John Day

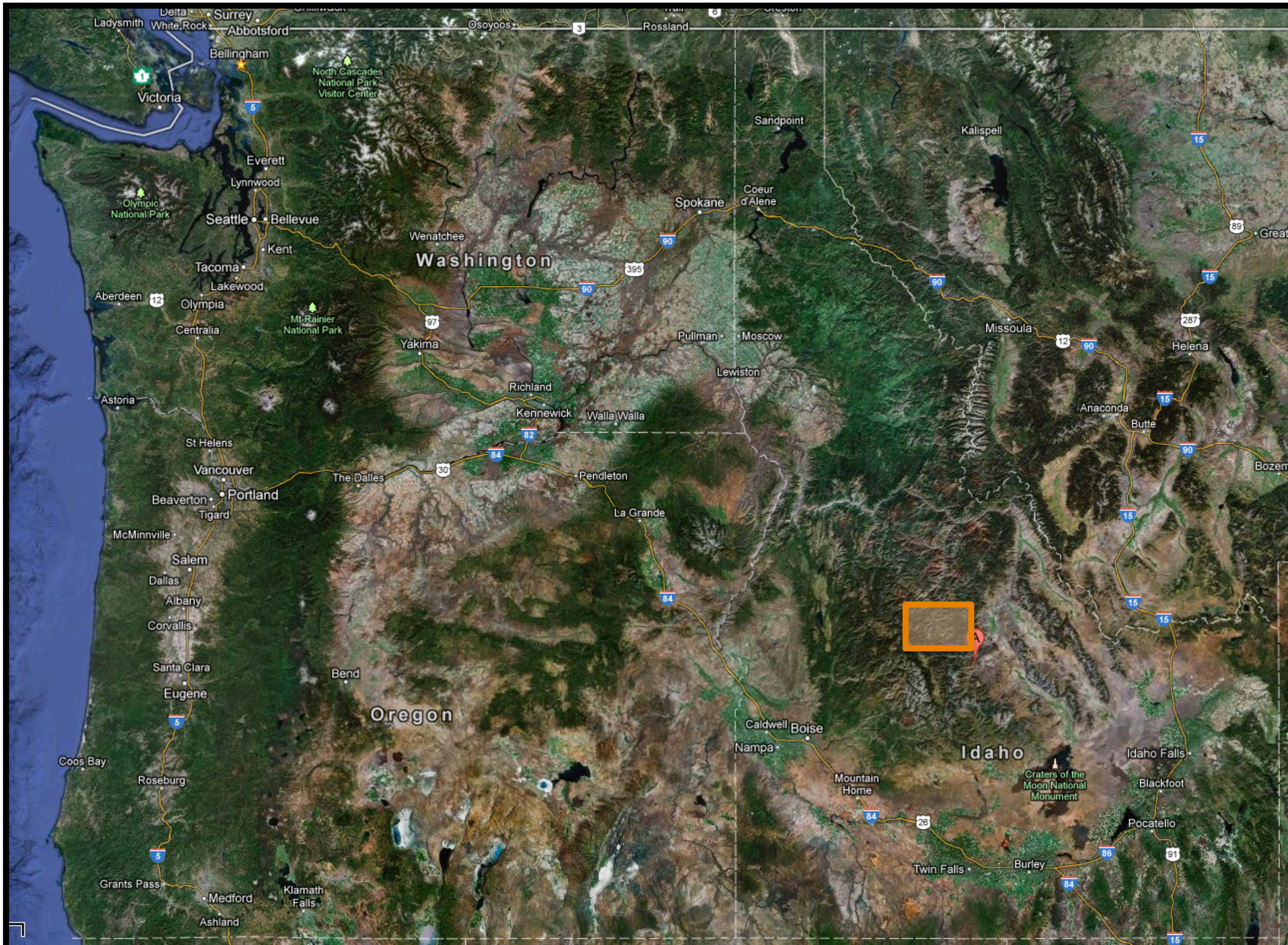
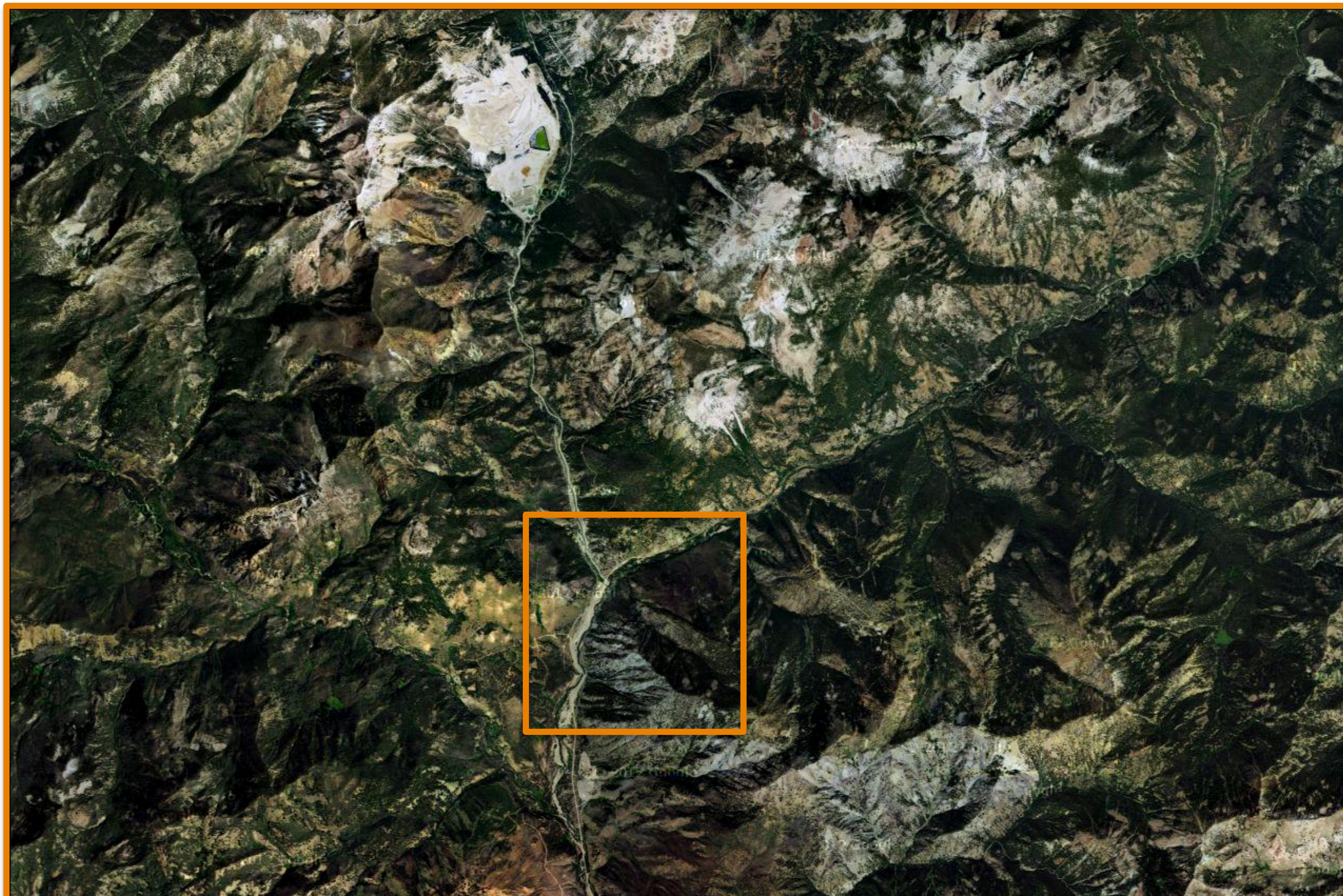


Image by Google



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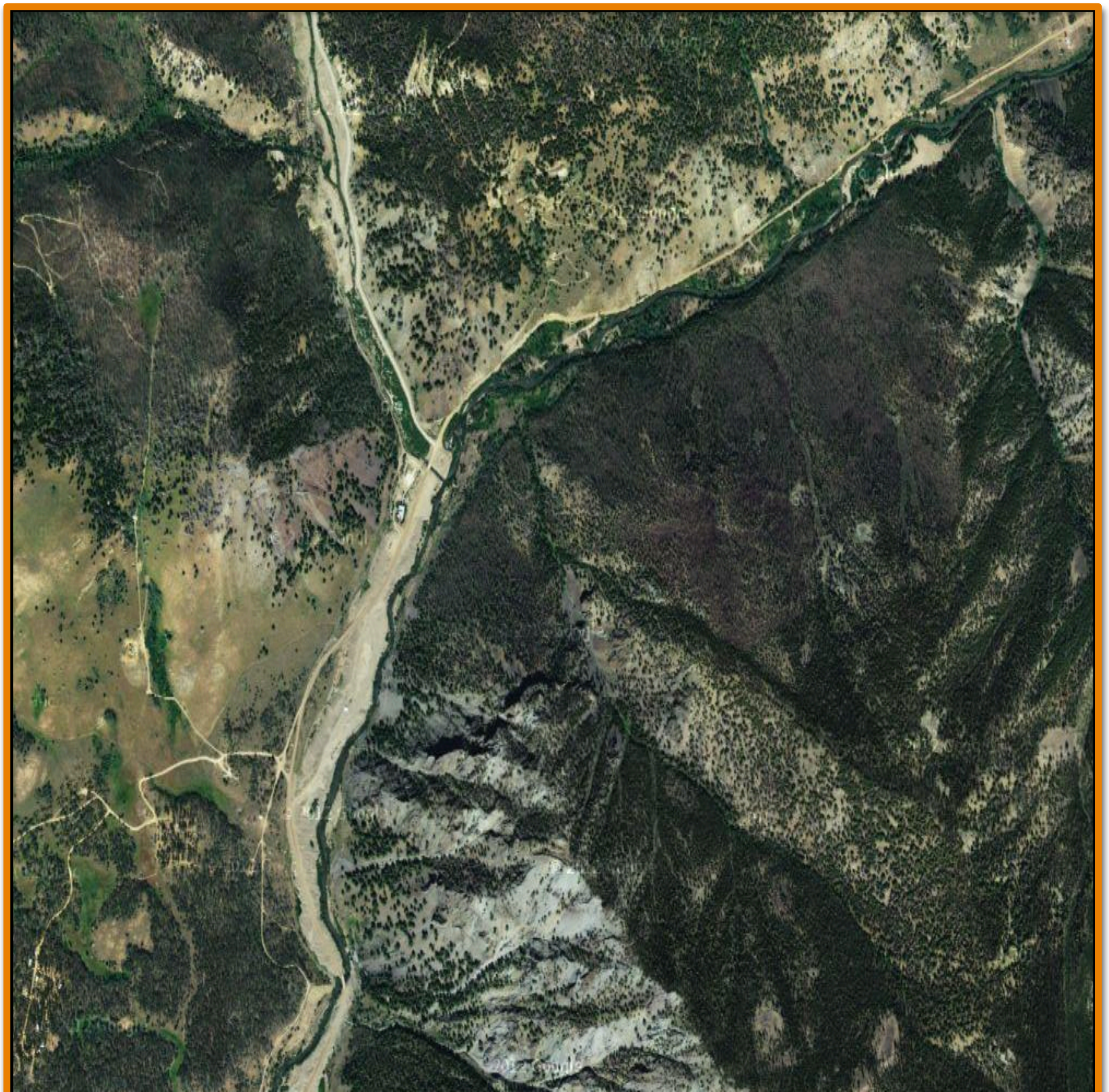

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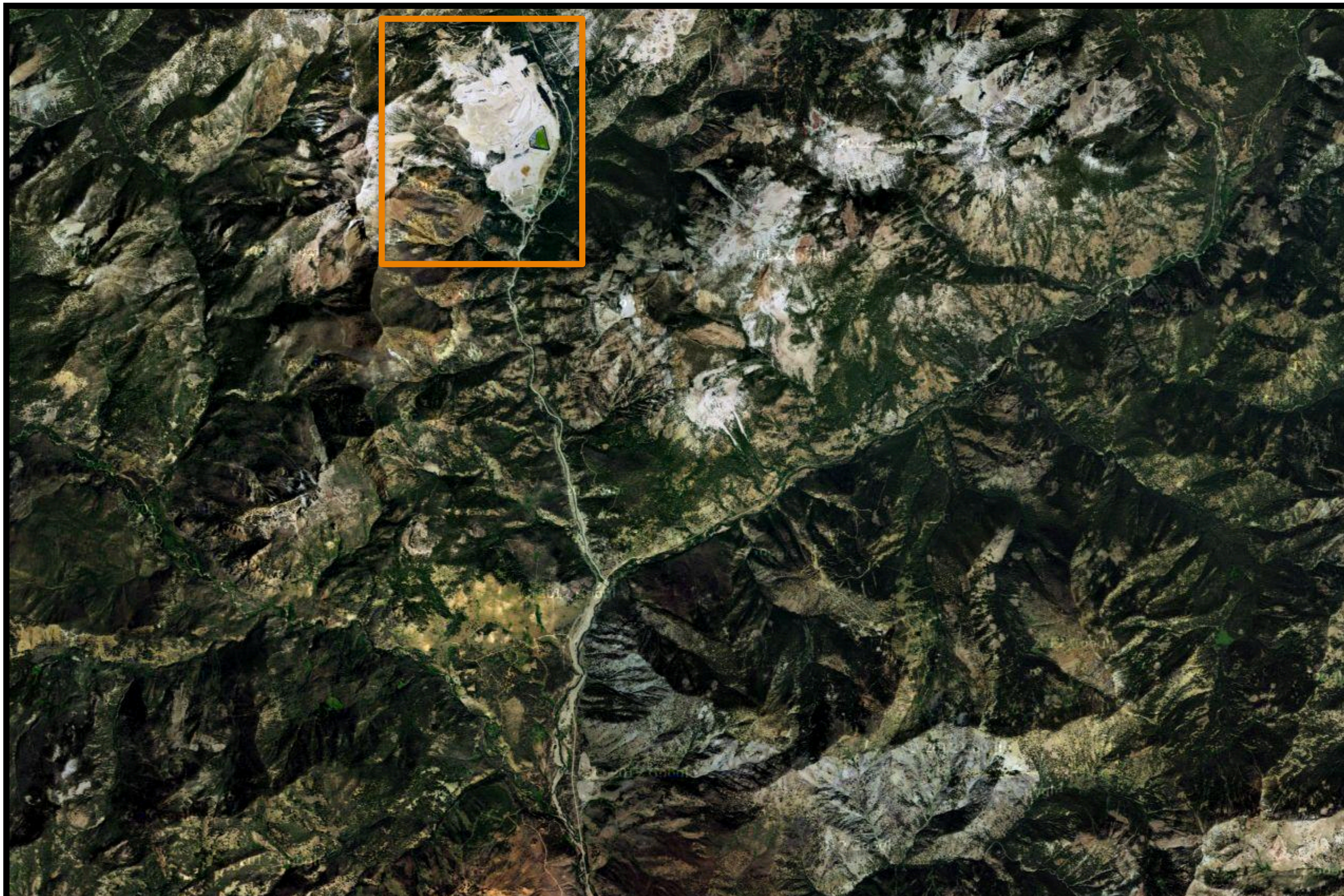
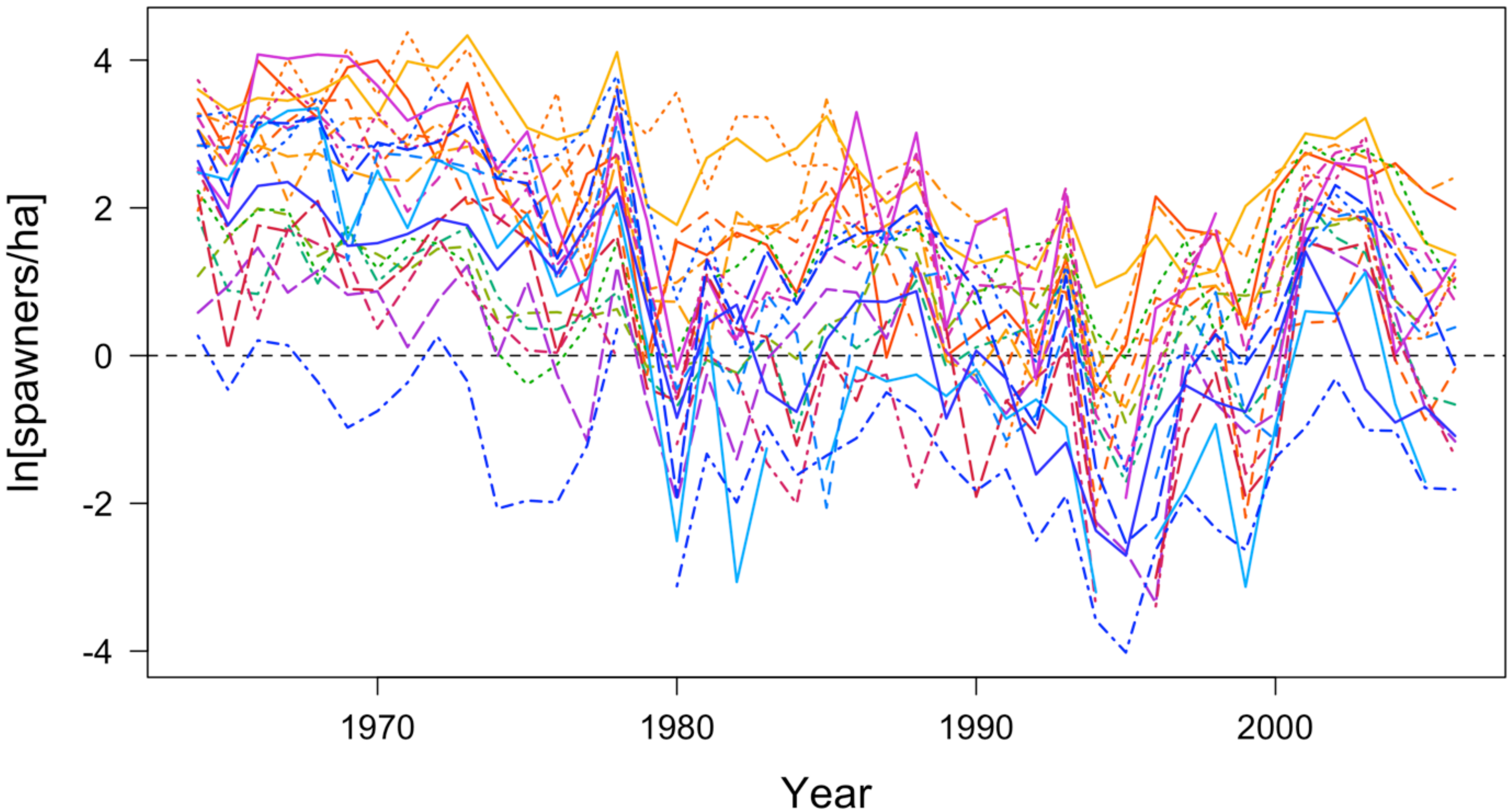
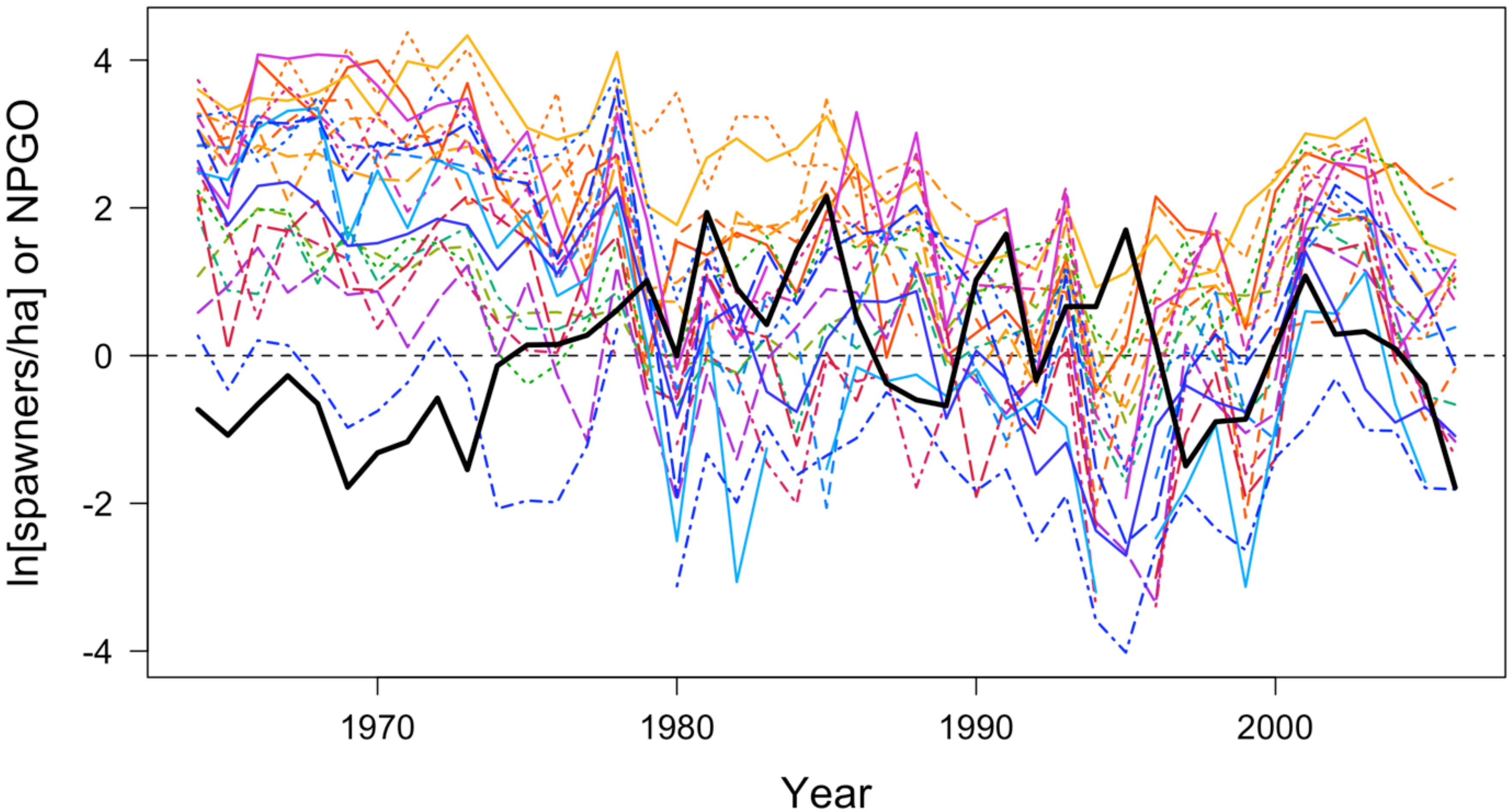


Photo by Jack Molan

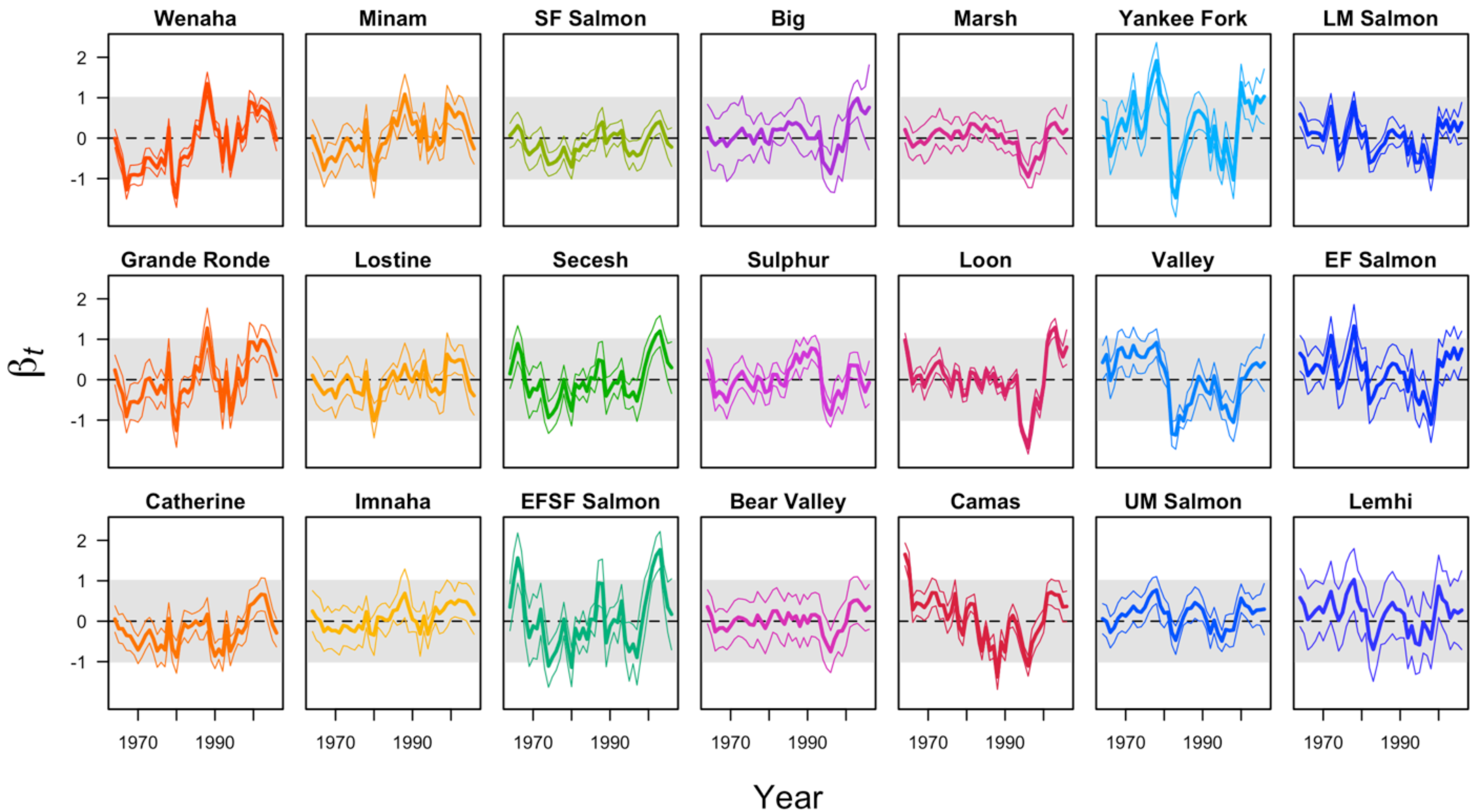
Time series of returns (Density)



Time series of returns & PDO

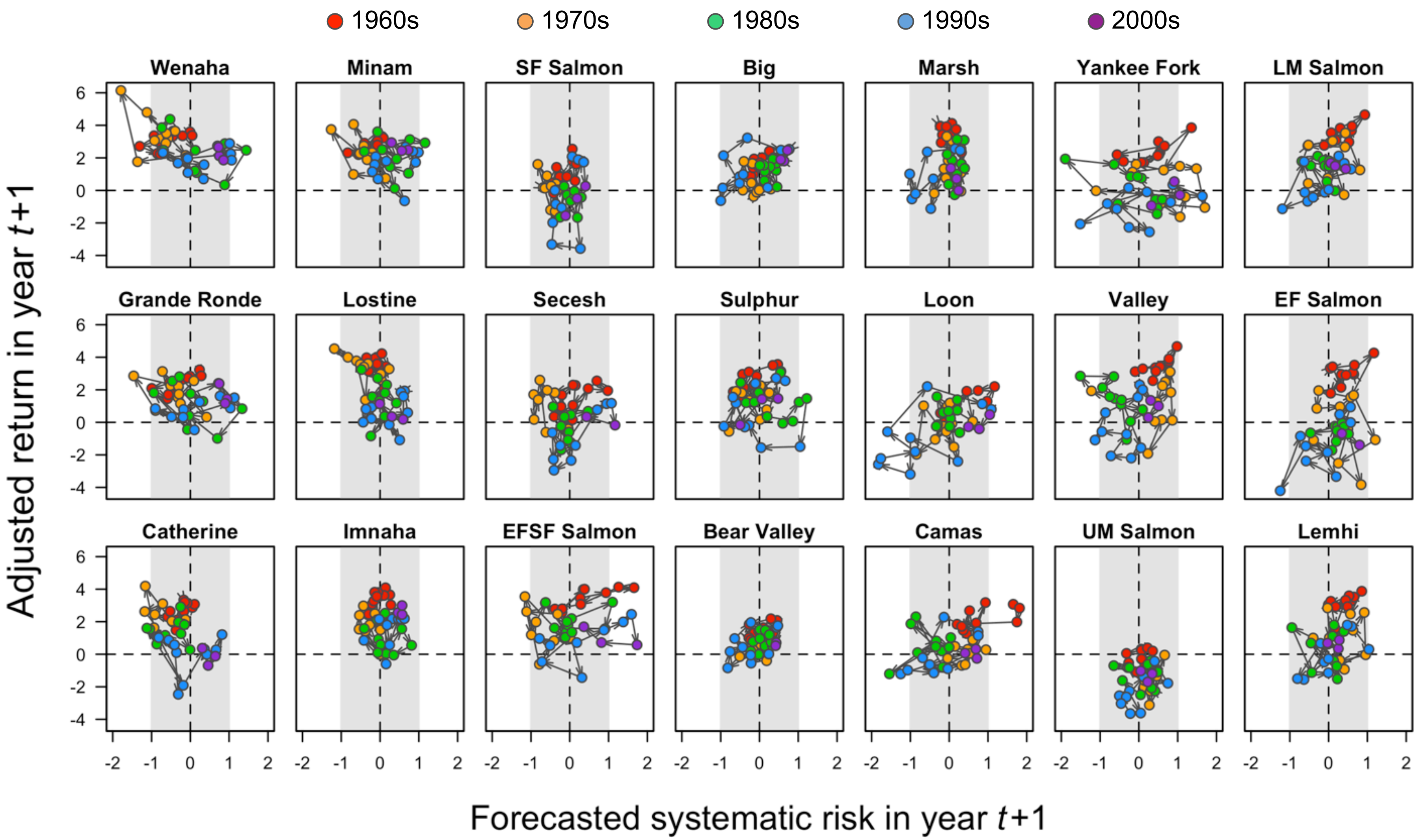


Results – time series of betas



Market = PDO

Results – CAPM



Market = PDO; *Risk-free* = 0