Forecasting with ARIMA models

Eli Holmes

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Forecasting with an ARIMA model

The basic idea of forecasting with an ARIMA model to estimate the parameters and forecast forward.

For example, let's say we want to forecast with a ARIMA(2,1,0) model with drift:

$$z_t = \mu + \beta_1 z_{t-1} + \beta_2 z_{t-2} + e_t$$

where $z_t = x_t - x_{t-1}$, the first difference.

`Arima()` would write this model:

$$(z_t - m) = \beta_1(z_{t-1} - m) + \beta_2(z_{t-2} - m) + e_t$$

The relationship between μ and m is $\mu = m(1 - \beta_1 - \beta_2)$.

Let's estimate the β's for this model from the anchovy data.

```
fit <- forecast::Arima(anchovyts, order=c(2,1,0), include.c
coef(fit)
```

```
##          ar1          ar2        drift
## -0.53850433 -0.44732522   0.05367062
```

```
mu <- coef(fit)[3]*(1-coef(fit)[1]-coef(fit)[2])
mu
```

```
##      drift
## 0.1065807
```

So we will forecast with this model:

$$z_t = 0.1065807 - 0.53850433z_{t-1} - 0.44732522z_{t-2} + e_t$$

To get our forecast for 1990, we do this

$$(x_{90} - x_{89}) = 0.106 - 0.538(x_{89} - x_{88}) - 0.447(x_{88} - x_{87})$$

Thus

$$x_{90} = x_{89} + 0.106 - 0.538(x_{89} - x_{88}) - 0.447(x_{88} - x_{87})$$

Here is R code to do that:

```
anchovyts[26]+mu+coef(fit)[1]*(anchovyts[26]-anchovyts[25])
  coef(fit)[2]*(anchovyts[25]-anchovyts[24])

## drift
## 9.962083
```

Forecasting with `forecast()`

`forecast(fit, h=h)` automates the forecast calculations for us and computes the upper and lower prediction intervals. Prediction intervals include uncertainty in parameter estimates plus the process error uncertainty.
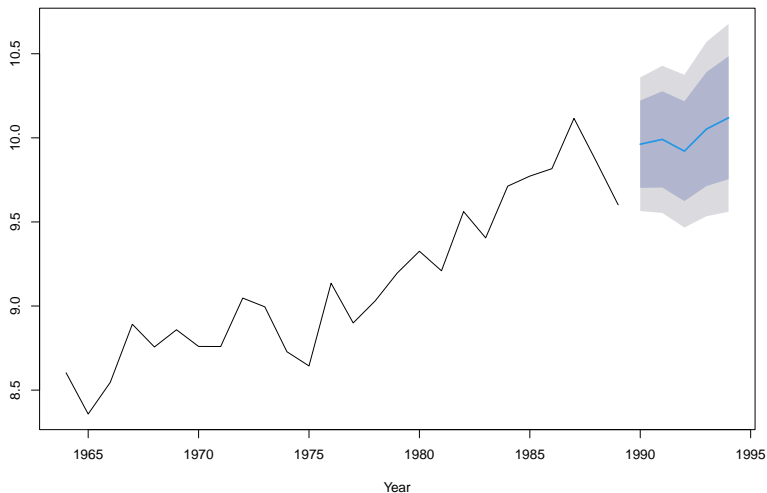
```
fr <- forecast::forecast(fit, h=5)
fr
```

```
##      Point Forecast    Lo 80     Hi 80     Lo 95     Hi 95
## 1990       9.962083 9.702309 10.22186 9.564793 10.35937
## 1991       9.990922 9.704819 10.27703 9.553365 10.42848
## 1992       9.920798 9.623984 10.21761 9.466861 10.37473
## 1993      10.052240 9.713327 10.39115 9.533917 10.57056
## 1994      10.119407 9.754101 10.48471 9.560719 10.67809
```

Plotting our forecasts

```
plot(fr, xlab="Year")
```

Forecasts from ARIMA(2,1,0) with drift

Missing values

Missing values are allowed for forecast::Arima(). We can produce forecasts with the same code.

```
anchovy.miss <- anchovyts
anchovy.miss[10:11] <- NA
anchovy.miss[20:21] <- NA
fit <- forecast::Arima(anchovy.miss, order=c(2,1,0), inclu
fr <- forecast::forecast(fit, h=5)
fr
```

```
##      Point Forecast    Lo 80    Hi 80    Lo 95    Hi 95
## 1990       9.938269 9.664479 10.21206 9.519543 10.35700
## 1991      10.014686 9.700961 10.32841 9.534885 10.49449
## 1992       9.924208 9.601147 10.24727 9.430129 10.41829
## 1993      10.029988 9.666069 10.39391 9.473421 10.58656
## 1994      10.128066 9.729066 10.52707 9.517848 10.73828
```

```
plot(fr)
```


Forecasts from ARIMA(2,1,0) with drift

Using auto.arima()

We can let forecast to select the ARIMA model:

```r
anchovy.miss <- anchovyts
anchovy.miss[10:11] <- NA
anchovy.miss[20:21] <- NA
fit <- forecast::auto.arima(anchovy.miss)
fit
```

```
## Series: anchovy.miss
## ARIMA(0,1,1) with drift
##
## Coefficients:
##           ma1    drift
##       -0.7240   0.0548
## s.e.   0.2283   0.0125
##
## sigma^2 = 0.04355:  log likelihood = 3.52
## AIC=-1.03   AICc=0.11   BIC=2.63
```

```
fr <- forecast::forecast(fit, h=5)
plot(fr)
```


Forecasts from ARIMA(0,1,1) with drift